Using AHP and Experimentation in the Design of Refrigerator Dispenser Lighting

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Abstract

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Translating the Voice of the Customer (VOC) into quantifiable technical targets, such as those required in a quality function deployment (QFD) matrix, is a difficult process. This paper examined two methods, Design of Experiments (DoE) and the Analytical Hierarchy Process (AHP), and explained why each of these methods, while providing powerful tools for their intended purposes, were not well suited for translating subjective customer data into technical targets. As a result, the author proposed a new method combining the relative comparison tools of the AHP as a front end with the existing and powerful analytical tools of DoEs. To validate the new tool, the author performed an experiment using a very subjective and innovative product feature, refrigerator dispenser cavity lighting. The result of the validation process provided proof that the new method, while raising some questions, effectively provides direction for technical targets and other valuable information concerning customer preferences.

Acknowledgement

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Introduction

All good design processes start with the voice of the customer (VOC). The VOC is data obtained directly from the customer that represents the customer's requirements or needs. One cannot underestimate the value in obtaining and understanding the VOC. Customer requirements give a product its purpose and therefore, are the reason a product exists. The better a product satisfies a customer need, the more desirable it will be. Obtaining and understanding the VOC should be the first step in any design process. Starting the design process with a poor set of customer requirements will likely produce a design that does not satisfy the customer's needs, and therefore, will be unsuitable. Unfortunately, obtaining and understanding the VOC can be a very subjective and difficult task.

Typically, engineers operate in an objective measurable world where physics is the common denominator. Units such as length, mass, and volume define an engineer's world. However, customers do not think in terms of length, mass, or volume. Instead, customers think in subjective units such as appearance, usability, or comfort, which are usually difficult to translate into objective units that can be compared, analyzed, or optimized. The translation of customer requirements into technical criteria is paramount to producing a design that the customer will desire.

The most common tool used to record the VOC and translate it into technical design characteristics is Quality Function Deployment (QFD). The American Supplier Institute (ASI) defines QFD as "a systematic process that helps companies quickly understand and integrate clients' needs into their products or services" (ASI). However, it can be argued that QFD alone does not provide a complete set of tools to obtain the best design direction but only acts as an environment to organize and communicate a design process, which is supported by many other tools. To populate the matrices in QFD, one must be able to use many tools. These tools help the engineer understand the ranking of customer requirements, correlations between customer requirements and technical characteristics, and setting targets for technical characteristics. As a result, one's ability to use and apply these tools affects one's capability to produce a design that satisfies the customer's needs.

This paper will explore a method that combines features from design of experiments (DoE) and the Analytical Hierarchy Process (AHP) to create a tool that can be used to discover what technical characteristics are most important to the customer and what levels should be achieved to establish the most positive impact on the customer. To test this new tool, an exercise will be conducted to gather customer information concerning a subjective and evolving feature on refrigerators; dispenser cavity lighting. The methods explored in this paper are at the very root of providing a good design.

Capturing the Voice of the Customer

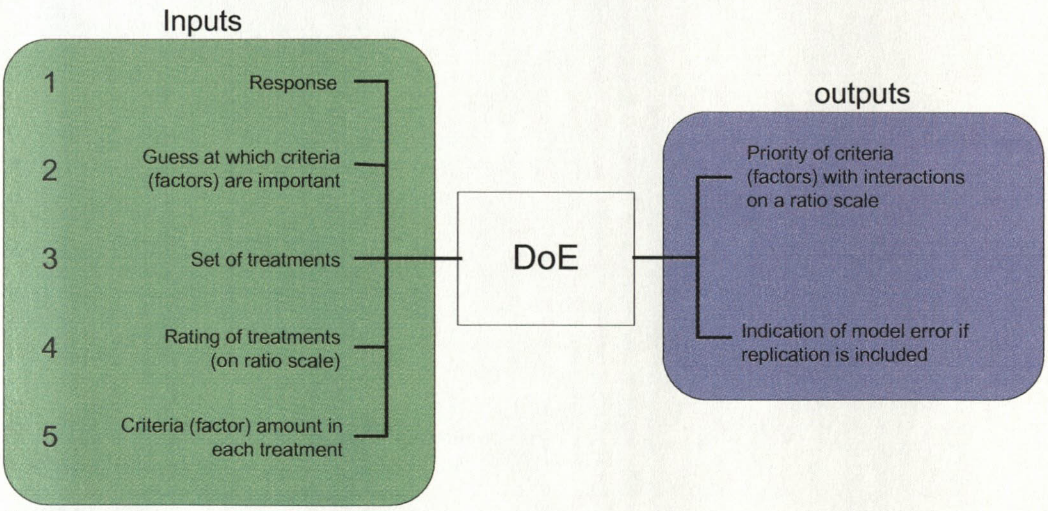
The web site iSixSigma (www.isixsigma.com) defines the voice of the customer as a "term used to describe the stated and unstated needs or requirements of the customer. The voice of the customer can be captured in a variety of ways: Direct discussion or interviews, surveys, focus groups, customer specifications, observation, warranty data, field reports, complaint logs, etc." iSixSigma goes on to also state that "This data is used to identify the quality attributes needed for a supplied component or material to incorporate in the process or product" (iSixSigma). From the definition above, one can surmise that the VOC is indispensable in discovering important product or process features. However, the author argues the VOC is not only useful for the identification of distinguishing or essential attributes, but is useful for the discovery and understanding of nearly all attributes.

Unfortunately, translating the VOC into useable and quantifiable data can be a challenge. For example, one can easily identify the most popular color for a car. This process may involve the customer selecting the most appealing car color from several pictures of cars. This data is easily translated into a technical description that describes the color. However, many attributes are not easily gleaned from the data. The customer can usually say they prefer 'A' to 'B', but commonly they cannot tell you why they selected 'A' to 'B' in terms of technical requirements. For instance, an engineer is trying to design a car that is fun to drive. Upon asking the customer 'what makes a car fun to drive?', they may not be able to tell you in terms that can be applied directly to the design of the car. If the information is not objective and quantifiable, it will be hard to incorporate into a car's technical specifications. To design a car that is 'fun to drive', the engineer first needs to understand which factors influence a customer's perception of how fun a car is to drive and secondly, the engineer needs to know the level settings for the influential factors. Suppose the engineer can change the engine's horsepower, the type of transmission, and the type of suspension. Are all of these factors important? If the engine's horsepower is important, what level should it be? 150hp? 200hp? Does the type of transmission matter and if it does, should it be a manual transmission or an automatic transmission? Does the type of suspension influence the customer's driving experience? To answer these questions, the engineer will need to conduct some type of experiment. The experiment should be capable of determining the customer's preferences for a set of treatments and additionally, it should be able to translate these preferences into technical requirements.

Design of Experiments

For translating customer preferences into technical requirements, many engineers will use a set of statistical tools known as Design of Experiments (DoE). Figure 1 is a simplified representation of the inputs and outputs required to conduct a DoE.

Figure 1 DoE inputs and outputs.



Source: Product work of the author.

A DoE requires five inputs to complete. First, the engineer needs to define a response. In the car experiment, the engineer might ask the customer to rate their driving experience on a scale of 1-to-10, with 1 representing no fun and a 10 representing the most fun. The response in this case, is a purely subjective rating by the customer.

Secondly, the engineer is required to supply a set of factors that may influence the customer's driving experience and in particular, the experience related to how fun the car is to drive. If the factors do not influence the driving experience, the analysis will show it. The engineer also needs to provide a set of alternatives, which are determined by the structure of the experiment. Alternatives in DoE terminology are referred to as treatments. Each treatment is composed of a combination of the factors at defined level settings.

Next, the engineer needs to collect the rating information from the customer. This rating system can be any numerical assignment that conforms to the defined scale and indicates the customer's preference. In the car experiment, one may use a 1-to-10 scale.

Lastly, the engineer will need to know how much content of each factor is in each treatment. If the data type is quantitative, the amount of each factor will be a measure representing the amount. If the data is a qualitative attribute data type, only the level is required. Examples of attribute data are no and yes, good and bad, and small, medium, or large. For example, treatment one may have an engine with 150hp (quantitative data), a manual transmission (attribute data), and suspension type 1 (attribute data). Furthermore, treatment two may have a 200hp engine, a manual transmission, and a suspension of type 2, and so on.

When the experiment is completed, the engineer will analyze the data and document the results. The results or outputs of the experiment are an indication of factor importance relative to its influence on the response. If the experiment is performed multiple times (replication), the results can also provide an indication of experimental error.

In summary, DoEs take as an input, the ratings of a set of treatments and produce a response, which when analyzed in the context of the experiment, yield a metric that indicates a factor's significance in affecting the value of the response. The DoE approach appears to be the perfect tool for translating a customer's opinions concerning a set of treatments into an indication of each factor's importance. Furthermore, the information from a DoE can be used to construct a model, which can be used to optimize the response. However, the collection of customer data is typically done using rating systems such as the 1-to-10 scale assumed in the previous example and similar scales such as the popular Likert scale. Rating scales suffer from a serious limitation when used in DoEs.

Data collection and measurement

When rating scales are used to collect data for inputs to experiments, it is common for an engineer to assume the scale is absolute. This assumption can cause serious errors in the results. This error is common because engineers are comfortable collecting data using absolute measurement systems, which utilize units such as temperature, kilograms, or voltage. Absolute implies that the measurement can be tied to a common standard or reference and if well established, will result in the same measurement value anywhere the measurement is made[1] (Saaty, 2000: 23). Anyone taking a common measurement using an absolute measurement system where standards are established uses the same standard reference, either directly or indirectly. For

[1] Absolute measurements also vary due to measurement error.

example, if one measures the length of an object, they may use meters. The meter is defined precisely and everyone uses the same standard. Therefore, the communication of data that was gathered using a well-established absolute measurement system is unhampered by a lack of standards. However, a lack of standards is not the main issue with a rating scale as used in this context because subjective measurements are expected to vary between respondents. There is a more covert and deceptive issue with the absolute assumption. Absolute measurement also implies that a standard is constant and does not change. When one is subjectively rating items, their internal standard for comparison is likely not established, therefore it will usually change as one progresses through the treatments. The standard used for the first treatment will likely be different for the second treatment and so on, as one's internal standard is redefined.

A good example of an internal standard drifting is when the respondent 'saves' some of the scale because they do not want to use the maximum value of the scale because future treatments may be much better. The author recently experienced this when watching a monster truck freestyle contest on television with his son. The contest used three judges and each monster truck was required to perform a series of freestyle maneuvers for a defined amount of time. If a contestant early in the lineup performed well, the commentators would often point out that the scores do not reflect the 'true score' because the judges are holding back in case a later performance is better. Therefore, it is rare for contestants scheduled early in the lineup to achieve high scores. The judges are allowing their standard to adjust during the contest. This results in the last contestant possessing a significant advantage over the earlier contestants because the judge's internal standard is well developed and the last contestant knows what this standard is. Of course, the opposite condition is also possible where the respondent assigns an early treatment a very high rating and the future treatments are assigned the near maximum or maximum scale value when in fact, the difference between the later treatments, and earlier treatment should be much greater.

One may argue that all treatments could be simultaneously presented to the customer for their evaluation. In this scenario, the customer would be allowed to review all of the treatments simultaneously and develop their internal standard. After their internal standard is sufficiently developed, the customer would assign a rating to each treatment. As stated previously, this type of measurement is called absolute measurement and works well when an established standard is available. However, establishing a subjective standard can be difficult. Moreover, in the author's experience, this type of experiment is rarely feasible. Experimental design is usually a tradeoff between knowledge gained and resources expended. In most situations, presenting the full gamut of treatments is resource prohibitive.

To better understand why the phenomenon of a drifting internal standard is undesirable, one requires an understanding of measurement and measurement scales. Wikipedia.org describes the process of measurement as "estimating the ratio of the magnitude of a quantity to the magnitude of a unit of the same type (length, time, mass, etc.). A measurement is the result of such a process, expressed as the product of a real number and a unit, where the real number is the estimated ratio. An example is 9 metres, which is an estimate of an object's length relative to a unit of length, the metre" (Wikipedia). Put more simply, measurement is the comparison of one measured or defined property to another measured property. When one uses a ruler, they compare a length on the ruler to the length of the object being measured. The point on the ruler where the measurement is taken is a measured property of another object transferred to the ruler. As stated previously, when a common standard for a reference exists, the measurement system is known as absolute. However, absolute measurement systems are not always available. For this reason and others, the data gathered using a measurement system possesses more or less information depending on a measurement property known as the measurement level. Assuming the wrong measurement level can result in serious errors.

Data Measurement

A system of measurement levels was first introduced by S.S. Stevens in the 1940s to define the set of statistical procedures that can be performed on the data. According to Stevens, measurement data can be classified into four basic levels; nominal, ordinal, interval, and ratio (Stevens, 1946: 103). As shown in Table 1, the amount of information contained in the data increases as one moves from nominal measurements to ratio measurements.

Table 1 Measurement levels.

Measurement level	Information
ratio	order, equal spacing, zero
interval	order, equal spacing
ordinal	order
nominal	category only

Source: Product work of the author.

Nominal level data contain the least amount of information. Measurements made using a nominal measurement system assign a category to each object measured. Religious preference, object color, and gender are examples of nominal measurements.

The next level up is ordinal. Ordinal data, as the name implies, contains information related to the order or the rank of the objects. Typically, information collected using a rating scale results in ordinal data. Furthermore, any information collected using a measurement system with a rating scale and a drifting internal standard will result in ordinal data.

Interval data contains order and an equal distance on the scale implies an equal distance between measurements. Therefore, the units on an interval scale are evenly spaced. An example of an interval scale is the Fahrenheit temperature scale. Many times, an ordinal scale is mistaken to be an interval scale.

The ratio scale possesses the characteristics of order and equal spacing like an interval scale and additionally, a ratio scale possesses a true zero. Ratio data sets contain the most information and are the most common scale used in the physical sciences. It is also important to note that just because a measurement scale is interval does not mean it cannot be converted to a ratio scale. For example, measurements taken using the Fahrenheit temperature scale, which are interval, can be converted to the Kelvin temperature scale, a ratio scale.

For example, assume one has collected the temperatures of ten items using four different measurement scales and each scale uses one of the four different levels of measurement. This data is shown in Table 2. The first column is the order the data was collected in and the four subsequent columns represent the same temperature data collected at different measurement levels.

Table 2 Temperature data.

Order collected	Nominal	Rank	Interval	Ratio
1	A	5	76.81	297.89
2	B	3	42.66	278.91
3	C	6	88.90	304.61
4	D	2	24.40	268.77
5	E	1	12.54	262.18
6	B	3	42.14	278.63
7	F	4	55.32	285.95
8	D	2	30.75	272.30
9	C	6	92.72	306.73
10	B	3	42.28	278.71

Source: Product work of the author.

The first measurements were collected as nominal data. In cases where the temperatures were close and the measurement device could not distinguish between the measurements, the data were grouped into one category. This grouping scheme created six different categories: A, B, C, D, E, and F. At the nominal measurement level, one can only count the number of items in each category. If desired, one can count the number of items in each category to create a histogram. For example, three temperatures were placed in the 'B' category.

The second group of measurements was collected as ordinal data using the same grouping scheme used to collect the nominal data. However, numbers are used to designate rank among the data. Collecting the data as ordinal data will allow one to determine which group is the hottest and the coldest. Additionally, ordinal data can be sorted. Table 3 contains the same data as Table 2 but the data is sorted from hottest to coldest.

Table 3 Temperature data sorted.

Order collected	Nominal	Rank	Interval	Ratio
5	E	1	12.54	262.18
4	D	2	24.40	268.77
8	D	2	30.75	272.30
6	B	3	42.14	278.63
10	B	3	42.28	278.71
2	B	3	42.66	278.91
7	F	4	55.32	285.95
1	A	5	76.81	297.89
3	C	6	88.90	304.61
9	C	6	92.72	306.73

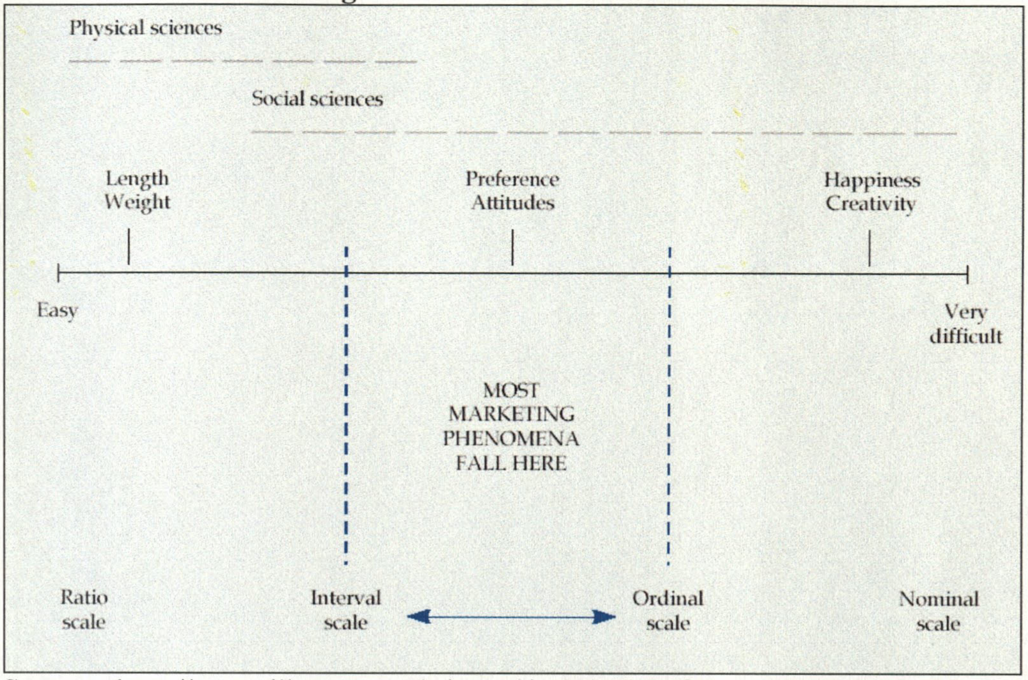
Source: Product work of the author.

The third group of measurements was collected as interval data using the Fahrenheit temperature scale. This data can be added and subtracted. For example, one can say that a certain temperature is a certain distance i.e. temperature, from another temperature. Ratios of the measurements with this data have no meaning. The fourth column from the left in Table 2 and Table 3 contains interval data.

The fourth set of data was collected as ratio data using the Kelvin temperature scale. Using this level of measurement, one can state that two measurements are a certain distance apart and additionally, one can express two temperatures as a ratio. For example, the temperature data in the first row (the fifth collected) and the fifth column (the ratio data) of Table 3 is 14% less than the temperature data in the ninth row (the third collected) of the same column.

As one can surmise from the above discussion, the collection of ratio data is the desired measurement level because it allows the greatest amount of mathematical operations and therefore, contains the most information. It is not surprising that ratio data is also the most common type of data collected in the physical sciences where absolute measurement systems are common. However, the collection of subjective data, like the data gathered from customers, is usually collected using rating scales like the 1-to-10 scale, the popular Likert scale, and others that typically produce ordinal data and occasionally interval data. The desirability of a ratio measurement system will normally be in conflict with the collection of customer data or marketing phenomena, as shown in Figure 2. Most data collected from marketing phenomena will be in-between the ordinal and interval measurement scales. Furthermore, without the knowledge that data are completely interval, they must be considered ordinal. An additional advantage of ratio data over other types is that the analysis is easier. Data analysis becomes more difficult as the scale type moves to the right in Figure 2 or toward a nominal scale.

Figure 2 Measurement scales.



Source: http://www.lib.uconn.edu/~punj/m35010.pdf.

Analytical Hierarchy Process

Does a tool exist that will allow the collection of subjective data, such as the data collected from customers, on a ratio scale? The answer is yes. The analytical hierarchy process (AHP) is a powerful tool for collecting subjective or 'soft data' and provides ratio data as an output. However, as the author will eventually explain, the inputs and outputs of the AHP do not fit the requirements of the desired process as presented by the author. Nonetheless, the author will make the case for using parts of the AHP to augment the DoE process. However, before discussing the creation of a new tool using parts of the AHP, one must have some understanding of the AHP. The next section will provide the reader with a brief summary of the AHP.

Thomas L. Saaty recognized the difficulty when making complex decisions and especially decisions that involve subjective criteria. In response to this, he developed the Analytical Hierarchy Process (AHP). The AHP is a multiple criteria decision analysis technique (MCDA) that can be used with subjective and objective criteria. The AHP uses paired comparisons of tangible or intangible alternatives to create a ratio scale of absolute numbers, which represent the priorities of the alternatives (Saaty, 2000). According to Saaty:

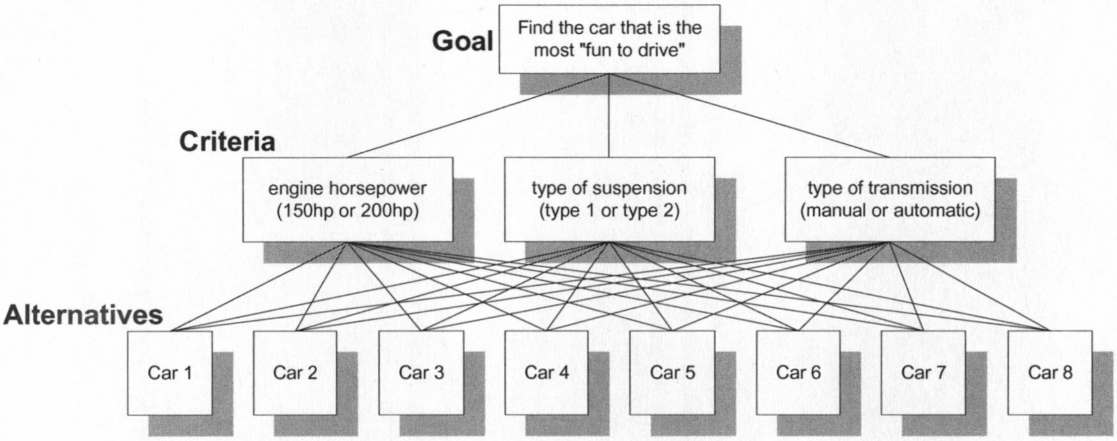
> "AHP is a method of breaking down a complex, unstructured situation into its component parts; arranging these parts, or variables, into a hierarchic order; assigning numerical values to subjective judgments on the relative importance of each variable; and synthesizing the judgments to determine which variables have the highest priority and should be acted upon to influence the outcome of the situation." (Saaty, 2001: 5).

The three primary functions of AHP are structuring complexity, measurement on a ratio scale, and synthesis (Forman and Gass, 1999).

Structuring Complexity (decomposition)

The AHP uses hierarchies to structure complexity into a more understandable format. "A hierarchy is a representation of a complex problem in a multilevel structure whose first level is the goal followed successively by levels of factors, criteria, sub-criteria, and so on down to a bottom level of alternatives" (Saaty, 2000: 94). "Arranging the goals, attributes, issues, and stakeholders in a hierarchy serves three purposes: It provides an overall view of the complex relationships inherent in the situation; it captures the spread of influence from the more important and general criteria to the less important ones; and it permits the decision maker to assess whether he or she is comparing issues of the same order of magnitude in weight or impact on the solution." (Saaty, 2000: 96). A simple hierarchy, which represents the car example used earlier in this paper, is shown in Figure 3. The graphical representation of a hierarchy is a powerful tool for representing complexity. At a glance, one can understand the relationships of goals to criteria and criteria to alternatives. Additionally, one can assess the likeness of alternatives within the same level.

Figure 3 Example hierarchy.

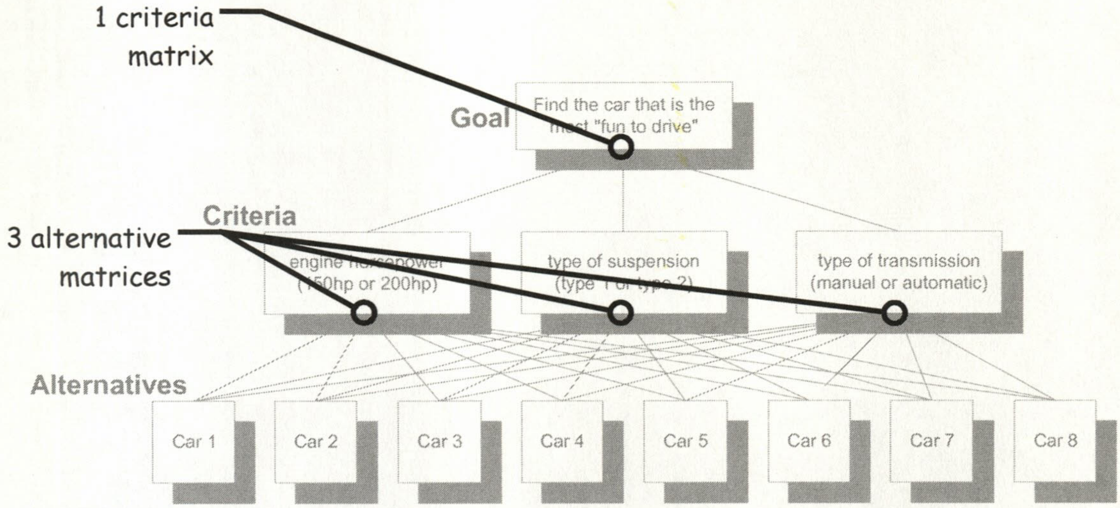


Source: Product work of the author.

Measurement on a Ratio Scale (comparative judgments)

The second function of AHP, measurement on a ratio scale, allows one to use a relative measurement system via a matrix of paired comparisons to derive a ratio scale of absolute numbers. The matrix of paired comparisons lists the criteria or alternatives as rows and columns of the matrix. One must complete four paired comparison matrices for the previous example shown in Figure 4.

Figure 4 Car example matrices.



Source: Product work of the author.

In each alternative matrix, one must ask how much more one alternative satisfies the goal over another in reference to a single criterion. In the criteria matrix, one must ask how important one criterion is over another in achieving the goal. For example, Figure 5 shows the form of the alternative matrices.

Figure 5 Alternative matrix form.

	car 1	car 2	car 3	car 4	car 5	car 6	car 7	car 8
car 1	1	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
car 2	$1/a_{12}$	1	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}
car 3	$1/a_{13}$	$1/a_{23}$	1	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}
car 4	$1/a_{14}$	$1/a_{24}$	$1/a_{34}$	1	a_{45}	a_{46}	a_{47}	a_{48}
car 5	$1/a_{15}$	$1/a_{25}$	$1/a_{35}$	$1/a_{45}$	1	a_{56}	a_{57}	a_{58}
car 6	$1/a_{16}$	$1/a_{26}$	$1/a_{36}$	$1/a_{46}$	$1/a_{56}$	1	a_{67}	a_{68}
car 7	$1/a_{17}$	$1/a_{27}$	$1/a_{37}$	$1/a_{47}$	$1/a_{57}$	$1/a_{67}$	1	a_{78}
car 8	$1/a_{18}$	$1/a_{28}$	$1/a_{38}$	$1/a_{48}$	$1/a_{58}$	$1/a_{68}$	$1/a_{78}$	1

Source: Product work of the author.

Each a_{ij} represents the perceived relative strength or dominance of i over j. For example, the customer may be asked to rate 'car 1' to 'car 2' in reference to the criterion 'engine horsepower'. The result of this comparison is the value a_{12}. The matrix of paired comparisons is a positive reciprocal matrix, where $a_{ij} = 1/a_{ji}$ and $a_{ij} = 1$ for i = j. Therefore, one only needs to complete the upper half of the matrix, which for eight alternatives requires one to make 28 paired comparisons. The 28 paired comparisons are highlighted in Figure 5. For the general case of N alternatives, one must complete $(N^2 - N)/2$ paired comparisons. The same method is used to complete the criteria matrix. However, there are only three criteria, resulting in a paired-comparison matrix for the criteria of order three.

When the customer compares the alternatives, a scale must be used to express the dominance of one alternative over the other. The AHP uses a 1-to-9 scale as shown in Table 4.

Table 4 The fundamental scale from Saaty.

Intensity of Importance	Definition	Explanation
1	**Equal Importance**	**Two activities contribute equally to the objective**
2	Weak	
3	**Moderate importance**	**Experience and judgment slightly favor one activity over another**
4	Moderate plus	
5	**Strong importance**	**Experience and judgment strongly favor one activity over another**
6	Strong plus	
7	**Very strong or demonstrated importance**	**An activity is favored very strongly over another; its dominance demonstrated in practice**
8	Very, very strong	
9	**Extreme importance**	**The evidence favoring one activity over another is of the highest possible order of affirmation**
Reciprocals of above	**If activity *i* has one of the above nonzero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i***	**A reasonable assumption**
Rationals	**Ratios arising from the scale**	**If consistency were to be forced by obtaining n numerical values to span the matrix**

Source: Saaty, 2000: 73.

Guidance for setting levels of stimuli for Saaty's 1-to-9 scale can be found in the work of Weber and Fechner, which states that in order to produce sensations that follow an arithmetic sequence, the stimulus must follow a geometric sequence (Saaty, 2000: 70-71). An additional justification of the 1-to-9 scale originates in the requirement to maintain consistency as one makes paired comparisons (Saaty, 2000: 72). If ideas or alternatives are introduced which are drastically different from existing ones, one's mind requires an adjustment in the way the old ideas or alternatives are rated. Saaty estimates the ratio of one's desire for consistency with the desire for inconsistency i.e. change, at about 9 (Saaty, 2001: 300). Therefore, consistency or the homogeneity of items must be considerably greater than the inconsistency or the non-homogeneity of items. If non-homogeneity is too great, adjustment of existing relationships or one's internal reference may be required. To compare homogeneous items, Saaty proposes that one should not need a scale that extends beyond nine (Saaty, 2000: 72). According to Saaty "The 1-9 scale is a simple scale that serves well" (Saaty, 2000: 72).

Not everyone agrees with this scale and some empirical evidence suggests that other scales may work better for the AHP (Barzilai, 2001, Ishizaka, 2007). Poyhonen, Hamalainen, and Salo have performed experiments to indicate that alternative numerical scales yield more accurate estimates than Saaty's usual 1-to-9 scale and that these alternative scales reduce the inconsistency of the comparison matrix (Poyhonen, 1997). However, Saaty's 1-to-9 scale has widespread use in the AHP. Furthermore, if the comparisons are obtained exclusively with verbal descriptions i.e. equal, moderate, strong, very strong, or extreme, the scale can be adjusted after the data collection has been completed.

If the magnitudes of two stimuli are too close together or too far apart to be represented by the 1-to-9 scale, people cannot accurately describe the differences. Saaty's solution is to group homogenous items in clusters and use pivots, alternatives that are common in adjacent clusters, to establish the relationship between clusters (Saaty, 2000: 72).

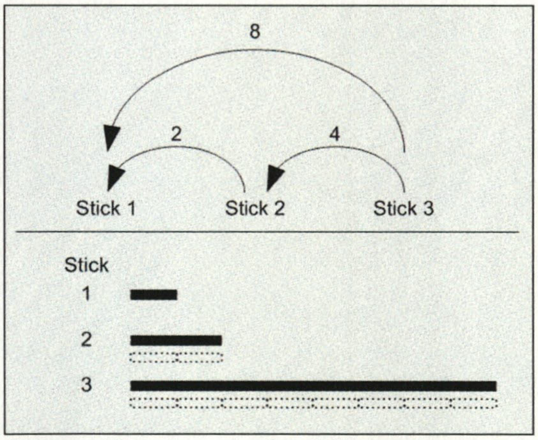
To obtain the customer's preference, alternatives are presented in pairs. The customer will be asked to indicate their preference as one of the verbal responses that correspond to the 1-to-9 scale. For example, using the car example presented earlier in Figure 3, if the customer drives car 1 and car 2 and is asked to rate car 1 compared to car 2, the customer might indicate that car 1 is strongly preferred to car 2. This results in $a_{12} = 7$. The opposite may also occur. The customer may indicate that they strongly prefer car 2 to car 1. This will result in $a_{12} = 1/7$. To complete the original comparison matrix, 28 such comparisons are required.

What is the maximum number of alternatives one can consider comparing before the number of comparisons becomes too large? Saaty recommends that "not many more than seven elements in a comparison scheme" (Saaty, 2000: 85). Saaty offers two explanations for this limit. The first reason is related to the consistency of measurements. The effects of inconsistency of the paired comparisons are distributed among the alternatives. When the alternatives are few, about seven, the priorities are relatively large, and a small inconsistency among the alternatives will have a negligible influence on the resulting priorities. However, if the number of alternatives are many i.e. much greater than seven, the resulting priorities will be small. Therefore, the impact of inconsistency on the priorities is much greater (Saaty, 2000: 85). The second explanation that Saaty offers is related to the brain's "limit on the identification of simultaneous events" (Saaty, 2000: 86).

One additional reason for not including too many alternatives stems from the limit of the evaluator's patience. One may loose interest in a long survey and in an effort to get finished quickly, offer inconsistent information. Therefore, the inconsistency of the information may be too great to be of any value.

Paired comparisons that are consistent, with one another, respect the property of transitivity. Transitivity is defined as $a_{ik} = a_{ij}a_{jk}$ for all i, j, and k where the subscripts denote the comparisons. Transitivity is shown graphically in Figure 6.

Figure 6 Transitivity.



Source: Product work of the author.

Figure 6 states that stick-two is two times longer than stick-one and stick-three is four times longer than stick-two. In order for the comparison of stick-one to stick-three to respect transitivity in relation to the comparisons of stick-one to stick-two and stick-two to stick-three, stick-three must be eight times longer than stick-one. If this relationship is true, these comparisons are consistent.

If all the measurements in a comparison matrix respect transitivity, then the matrix is said to be consistent. AHP does not require a consistent matrix but only a near consistent matrix (Saaty, 2000: 59). As will be shown, AHP offers a metric to measure inconsistency and a recommended upper limit that when exceeded, implies that the measurements may need to be reevaluated.

Consistency in a comparison matrix is desired. However, some inconsistency is expected. As one makes comparisons of new alternatives, his strategy for rating the old alternatives will change slightly. If the inconsistency of the comparison matrix is too great, the alternatives do not possess enough homogeneity or the evaluator needs to reconsider their earlier comparisons.

Synthesis of Priorities

After one completes all of the paired comparisons, an absolute ratio scale in the form of a priority vector must be obtained from each matrix of paired-comparisons. To derive the priority vector for each matrix, Saaty recommends using the principal eigenvector w. The principal eigenvector can be obtained by solving the equation $Aw = \lambda_{\max} w$ or $(A - \lambda_{\max} I)w = 0$ where A is the matrix, w is the principal eigenvector, and $\lambda_{\max}$ is the maximum eigenvalue. w is an n-vector $(v_1, v_2, \cdots, v_n)$, where each v_n represents an estimate of the n^{th} item's priority or dominance over the other items. $\lambda_{\max}$ is used as a gauge of a paired-comparison matrix's consistency. The proofs for the above claims can be found in Saaty (Saaty, 2000: 77-83).

In practice, the principal eigenvector, w, can be obtained by evaluating $\lim_{k \to \infty} \frac{A^k e}{e^T A^k e} = cw$, where A is the positive comparison matrix, $e = (1,1,\ldots,)^T$, and c is some constant (Saaty, 2000: 78-79). This quantity is solved numerically. As the iterations progress, 'cw' converges and the process can be terminated when the change in 'cw', the eigenvector times a constant, is less than some predetermined value. One can also use the geometric mean to produce the priority vector. However, Saaty recommends against this method since for matrices larger than three, the geometric mean can give incorrect results (Saaty, 2001: 84).

As mentioned earlier, the previous car example results in four paired-comparison matrices as shown in Table 5. From these four paired-comparison matrices, one will obtain four priority vectors, one for each matrix.

Table 5 Paired comparison matrices.

Criteria
Alternatives under the criteria 'engine horsepower'
Alternatives under the criteria 'type of suspension'
Alternatives under the criteria 'type of transmission'

Source: Product work of the author.

To arrive at a vector that rates the overall importance of the alternatives toward achieving the goal, the individual priority vectors must be combined into an overall priority vector. AHP offers the following two methods to obtain an overall priority vector: the distributive mode and the ideal mode. The difference in the two modes is how the priority vectors representing the alternatives with respect to each criterion are weighted.

In the distributive mode, each priority vector for the alternative comparison matrices and the criteria matrix is normalized by its sum. One then must combine these vectors into an overall priority vector. This is done by multiplying each alternative priority vector by its respective criterion priority, then adding the resulting priority vectors. The resulting priority vector has a dimension equal to the number of alternatives.

The ideal mode is very similar to the distributive mode, except each priority vector for the alternative comparison matrices is divided by its largest priority. Just as in the distributive mode, the priority vector from the criteria matrix is normalized by its sum. The priority vectors are combined using the same method used in the distributive mode. However, when synthesizing priorities in the ideal mode, one must complete the synthesis by normalizing the resulting vector by its sum.

One must choose which mode best fits the decision to be made. If one wants the choice to be independent of the number of alternatives, the ideal mode is best. In the ideal mode, the addition of future alternatives will not allow rank reversal. If the decision needs to allow for the influence of future alternatives and potential rank reversal, then the distributive mode is best. To illustrate the process, Tables 6 and 7 contain fictitious data based on the hierarchy shown in Figure 3. Table 6 contains the relative comparison matrix generated for the criteria. Each entry in the upper half of the matrix represents the dominance of one criterion over another in reference to achieving the goal. For example, the value at the intersection of row one and column two states that the criteria of 'engine horsepower' is strongly to very-strongly more important than 'type of suspension' for maximizing the goal of 'fun to drive'. Of course, this preference is translated into a six according to Saaty's 1-to-9 scale. The diagonal of the matrix is the comparison of an item to itself, which is always one, and the bottom half of the matrix is the reciprocal value of the top half of the matrix. To obtain the priority vector for the criteria, one must find the eigenvector and normalize the resulting value. The priority vector and the resulting maximum eigenvalue are displayed to the right of the comparison matrix. Each of the alternative matrices is shown in Table 7. For each entry in the upper half of an alternative's comparison matrix, one must ask how well does one alternative satisfy the goal compared to a competing alternative in reference to a particular criterion. The comparison matrices are also positive reciprocal matrices with diagonal values of one, therefore one only needs to complete the upper half of each matrix. To obtain the local priorities for the alternatives one must find the eigenvectors and normalize the results. The local priorities and the resulting eigenvalues are shown to the right of each alternative comparison matrix. After the priority vector for the criteria and the local priority vector for each alternative matrix are found, one can proceed with finding the overall priority vector. As stated previously, the overall priority can be obtained by using the distributive mode or the ideal mode. To find the overall priority vectors using the distributive method, the associated weight of each criterion must be multiplied times the local priority vector for each alternative. The result of this operation is the set of global priority vectors. After the global priority vectors are found, the overall priority vector is found by adding the global priority vectors. Table 8 shows the values obtained for this example using the distributive mode. To find the overall priority vector using the ideal mode, one follows a similar process. However, the local priorities are divided by their largest value. This operation requires the overall priority vector to be normalized. Table 9 shows the values obtained for this example using the ideal mode.

Another important result found in the process is the eigenvalue. The eigenvalue is used as a gauge of inconsistency. As stated previously, the principal eigenvector is obtained by solving the equation $Aw = \lambda_{\max} w$ or $(A - \lambda_{\max} I)w = 0$ where, A is the matrix, w is the principal eigenvector, and $\lambda_{\max}$ is the maximum eigenvalue. In practice, the eigenvalue is obtained by multiplying the priority vector, which is the equivalent of the eigenvector times a constant, times the vector obtained by creating row sums from the original comparison matrix. Each value in the row sum vector is obtained by solving the equation $v_j = \sum_{i=1}^{m} a_{ij}$ where m is the number of rows in the comparison matrix, i is the row number, and j is the column number. The eigenvalue is used to calculate the consistency index, which in turn is used to calculate a consistency ratio. The consistency index (C.I.) is calculated by solving $C.I. = (\lambda_{\max} - n)/(n-1)$ where, n is the number of rows or columns in the comparison matrix (the matrix is always square). Furthermore, the consistency ratio (C.R.) is found by forming the ratio of the consistency index to an average random consistency index (R.I.), $C.R. = C.I./R.I.$ The R.I. values used by Saaty are shown is Table 10 (Saaty, 2000: 84). The consistency ratio is a measure of inconsistency. A greater consistency ratio indicates a more inconsistent comparison matrix. How much inconsistency can be tolerated? Saaty states "Inconsistency may be thought of as an adjustment needed to improve the consistency of the comparisons. But the adjustment should not be as large as the judgment itself, nor so small that using it is of no consequence" (Saaty, 2000: 84-85). Saaty recommends an inconsistency or consistency ratio of no greater than 10% or 0.10 (Saaty, 2000: 85). If the inconsistency is greater than 10% for the comparison matrix, one should reevaluate the judgments or consider regrouping the items to form a more homogonous group.

Table 6 Criteria matrix.

	Criteria
A	engine horsepower
B	type of suspension
C	type of transmission

	A	**B**	**C**	**Priority vector**		
A	1.00	6.00	3.00	0.64	λ_{max}	3.05
B	0.17	1.00	0.25	0.09	**C.I.**	0.03
C	0.33	4.00	1.00	0.27	**C.R.**	0.05

Source: Product work of the author.

Table 7 Alternative matrices.

engine horsepower

	1	2	3	4	5	6	7	8	Priority vector		
1	1.00	3.00	4.00	0.50	2.00	1.00	6.00	7.00	0.201	**λ_{max}**	8.020
2	0.33	1.00	1.33	0.17	0.67	0.33	2.00	2.33	0.123	**C.I.**	0.003
3	0.25	0.75	1.00	0.13	0.50	0.25	1.50	1.75	0.162	**C.R.**	0.002
4	2.00	6.00	8.00	1.00	4.00	2.00	9.00	9.00	0.174		
5	0.50	1.50	2.00	0.25	1.00	0.50	3.00	3.50	0.086		
6	1.00	3.00	4.00	0.50	2.00	1.00	6.00	7.00	0.054		
7	0.17	0.50	0.67	0.11	0.33	0.17	1.00	1.17	0.093		
8	0.14	0.43	0.57	0.11	0.29	0.14	0.86	1.00	0.107		

type of suspension

	1	2	3	4	5	6	7	8	Priority vector		
1	1.00	2.00	3.00	0.50	0.33	3.00	1.00	0.20	0.079	**λ_{max}**	8.103
2	0.50	1.00	1.50	0.25	0.17	1.50	0.50	0.11	0.040	**C.I.**	0.015
3	0.33	0.67	1.00	0.17	0.11	1.00	0.33	0.11	0.028	**C.R.**	0.010
4	2.00	4.00	6.00	1.00	1.50	6.00	2.00	0.40	0.180		
5	3.00	6.00	9.00	0.67	1.00	9.00	3.00	0.60	0.218		
6	0.33	0.67	1.00	0.17	0.11	1.00	0.33	0.11	0.028		
7	1.00	2.00	3.00	0.50	0.33	3.00	1.00	0.20	0.079		
8	5.00	9.00	9.00	2.50	1.67	9.00	5.00	1.00	0.348		

type of transmission

	1	2	3	4	5	6	7	8	Priority vector		
1	1.00	2.00	3.00	0.20	2.00	0.14	5.00	2.00	0.085	**λ_{max}**	8.187
2	0.50	1.00	1.50	0.11	1.00	0.11	2.50	1.00	0.045	**C.I.**	0.027
3	0.33	0.67	1.00	0.11	0.67	0.11	1.67	0.67	0.033	**C.R.**	0.019
4	5.00	9.00	9.00	1.00	9.00	0.71	9.00	9.00	0.337		
5	0.50	1.00	1.50	0.11	1.00	0.11	2.50	1.00	0.045		
6	7.00	9.00	9.00	1.40	9.00	1.00	9.00	9.00	0.387		
7	0.20	0.40	0.60	0.11	0.40	0.11	1.00	0.40	0.024		
8	0.50	1.00	1.50	0.11	1.00	0.11	2.50	1.00	0.045		

Source: Product work of the author.

Table 8 Distributive mode.

A	B	C				
0.64	0.09	0.27				
Local priority vectors			Global priority vectors			Overall priority vector
0.201	0.079	0.085	0.129	0.007	0.023	0.16
0.123	0.040	0.045	0.079	0.003	0.012	0.09
0.162	0.028	0.033	0.104	0.002	0.009	0.12
0.174	0.180	0.337	0.112	0.015	0.091	0.22
0.086	0.218	0.045	0.056	0.019	0.012	0.09
0.054	0.028	0.387	0.035	0.002	0.105	0.14
0.093	0.079	0.024	0.060	0.007	0.006	0.07
0.107	0.348	0.045	0.069	0.030	0.012	0.11

Source: Product work of the author.

Table 9 Ideal mode.

A	B	C					
0.64	0.09	0.27					
Local priority vectors			Global priority vectors				Overall priority vector
1.000	0.227	0.219	0.644	0.019	0.059	0.72	0.17
0.614	0.115	0.115	0.395	0.010	0.031	0.44	0.11
0.805	0.081	0.085	0.518	0.007	0.023	0.55	0.13
0.868	0.518	0.869	0.559	0.044	0.235	0.84	0.20
0.429	0.627	0.115	0.276	0.053	0.031	0.36	0.09
0.268	0.081	1.000	0.172	0.007	0.271	0.45	0.11
0.461	0.227	0.061	0.297	0.019	0.017	0.33	0.08
0.533	1.000	0.115	0.343	0.085	0.031	0.46	0.11

Source: Product work of the author.

Table 10 Random consistency index at various values of n.

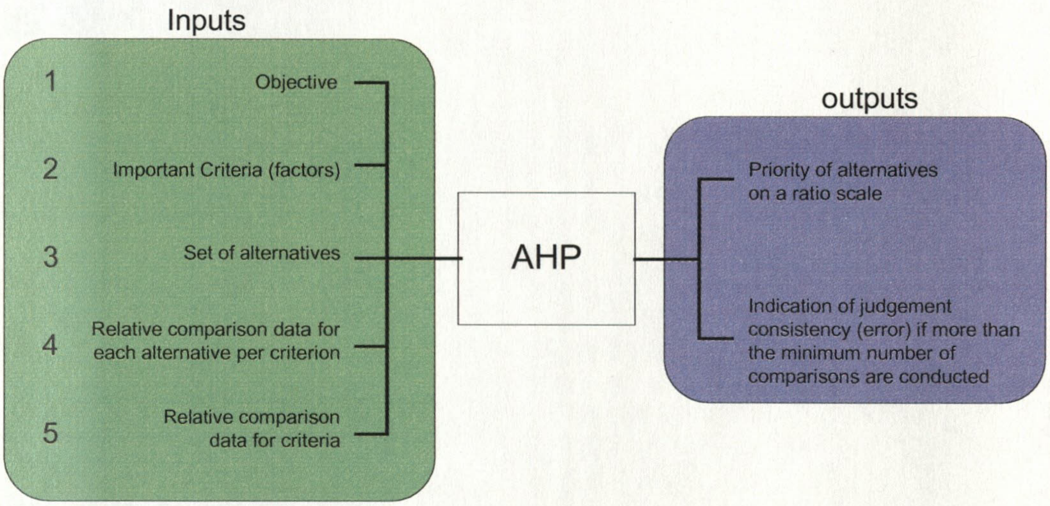
n	1	2	3	4	5	6	7	8	9	10
R.I.	0.00	0.00	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49
n	11	12	13	14	15					
R.I.	1.51	1.54	1.56	1.57	1.58					

Source: Saaty, 2000: 84.

Using AHP to collect Customer Data

Multicriteria decision making, also known as multicriteria decision analysis, as the name implies, is making a decision when two or more criteria exists and the criteria are usually conflicting. The AHP is a powerful tool. However, as the author mentioned previously, the inputs and outputs of the AHP process do not align with the requirements of a tool needed to learn about technical criteria.

Figure 7 Inputs and outputs of the AHP.



Source: Product work of the author.

One can see the inputs and outputs for the AHP in Figure 7. The inputs to the AHP require an intimate knowledge of the criteria and the alternatives relative to each criterion. If one's understanding of the criteria is not sufficient, the outcome of the AHP may not represent one's true or best choice. Typically, customers cannot supply the technical details, i.e. criteria, of why they like one alternative over another. The main output of the AHP is a priority vector representing the importance of the alternatives. However, customers can usually supply this information.

The AHP obviously does not supply the necessary outputs and requires inputs that usually cannot be provided by the customer. However, the AHP provides a very applicable approach to collecting customer data with the use of relative comparisons and the eigenvector method to arrive at a priority vector. To collect customer data, it seems that a combination of different tools might be most effective.

Combining AHP with DoE

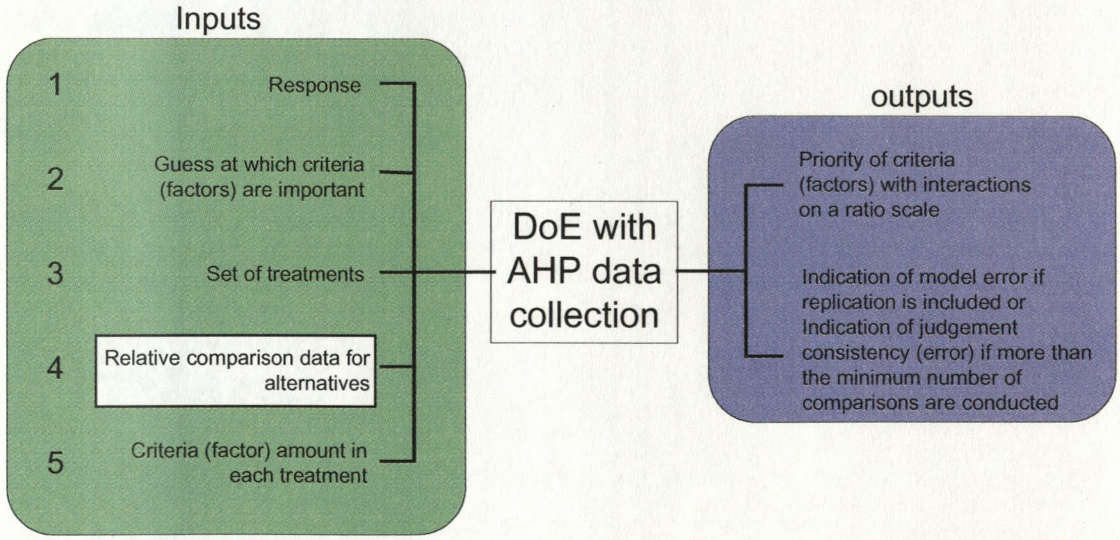
To review, a tool is desired that will accept subjective customer input and give the importance of the criteria for the various alternatives. From the previous discussion, one can surmise that the AHP will not suffice as a tool to learn about the criteria driving a customer preference when the customer knows very little about what drives their decisions. A customer knowing what they like but not being able to describe why they like it is common when one needs to understand technical characteristics such as those needed by engineers in product design.

One can also surmise from the previous discussion that DoEs, as previously described, do not provide a suitable method to collect customer data due to the typical rating scales used to collect customer data do not contain sufficient information. However, one may ask 'are there sections of the AHP method that can be combined with the DoE method to produce a tool that can accept subjective data and yield a rating of the criteria or factors'?

One of the AHP's many strengths is its ability to take subjective customer input in the form of a relative comparison matrix, obtained using paired comparisons of alternatives, and transform it into a priority vector of absolute ratio data. The author proposes that the ratio data obtained by using the paired-comparison method of AHP can be analyzed as a DoE to understand the criteria driving a customer's decision. Figure 8 is a graphic illustrating the use of the AHP as a 'front end' to a DoE. There are only two changes to the DoE process (one can reference Figure 1 for a graphic of DoE inputs and outputs). The main change is the substitution of relative comparison data in place of ratio data. The substitution of relative comparison data for ratio data allows one to collect subjective input. However, as will be discussed, it also restricts the number of treatments.

The other change to the DoE process is the addition of an indicator of consistency if redundant comparisons are included in the paired-comparison matrix.

Figure 8 DoE with AHP data collection.



Source: Product work of the author.

To use such a process, one first needs a tool to collect the raw data from the customer. A common tool used to collect customer data is a survey. "A survey is a systematic method of collecting information from a selected group of people by asking a series of questions" (Houston: 2). The survey should ask questions that elicit judgments regarding paired-comparisons from a paired-comparison matrix using Saaty's 1-to-9 scale. The matrix shall express dominance of alternatives relative to other alternatives in reference to a goal. For instance, in the car example used earlier, one would setup a paired-comparison matrix of alternatives with the intent of finding the alternative that is the most 'fun to drive'. The process would involve the customer being shown several pairs of alternatives and in each case; the customer would express their preference as the dominance of one car or alternative over another with respect to the goal of 'fun to drive'. The paired-comparisons are represented in the matrix shown in Figure 9.

Figure 9 Comparison matrix.

	car 1	car 2	car 3	car 4	car 5	car 6	car 7	car 8
car 1	1	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
car 2	$1/a_{12}$	1	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}
car 3	$1/a_{13}$	$1/a_{23}$	1	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}
car 4	$1/a_{14}$	$1/a_{24}$	$1/a_{34}$	1	a_{45}	a_{46}	a_{47}	a_{48}
car 5	$1/a_{15}$	$1/a_{25}$	$1/a_{35}$	$1/a_{45}$	1	a_{56}	a_{57}	a_{58}
car 6	$1/a_{16}$	$1/a_{26}$	$1/a_{36}$	$1/a_{46}$	$1/a_{56}$	1	a_{67}	a_{68}
car 7	$1/a_{17}$	$1/a_{27}$	$1/a_{37}$	$1/a_{47}$	$1/a_{57}$	$1/a_{67}$	1	a_{78}
car 8	$1/a_{18}$	$1/a_{28}$	$1/a_{38}$	$1/a_{48}$	$1/a_{58}$	$1/a_{68}$	$1/a_{78}$	1

Source: Product work of the author.

As previously discussed, each a_{ij} represents the perceived relative strength or dominance of i over j. To complete the matrix, one needs to gather $(N^2 - N)/2$ or 28 paired-comparisons. An important distinction is the dominance of one car over another is expressed relative to the goal without regard for any specific criterion as done in the

AHP. For example, the customer may be asked to rate 'car 1' to 'car 2' in regard to the goal of 'fun to drive'. The result of this comparison is the value a_{12}. As in the AHP, after the comparison matrix is completed, one uses the eigenvector method to arrive at a priority vector and an eigenvalue, which is used to generate an indicator of consistency.

The resulting priority vector is an absolute scale of ratio data that can be analyzed using DoE techniques. The results of the DoE analysis will indicate a factor or criterion's importance in determining the goal or response, which in this example is 'fun to drive'.

Maximum number of experimental factors

How many factors or criteria can be included in an experiment? Saaty recommends "not many more than seven elements in a comparison scheme" (Saaty, 85, 2000). The above example includes eight treatments, which in a 2-level full factorial design is three factors. Four treatment experiments with two factors have limited value. However, one can include more factors by using fractional factorial designs if confounding can be tolerated. Confounding is the aliasing of the main effects and interaction effects. For example, four factors can be included if one can tolerate the main effects confounding with three-way interactions. When a main effect is confounded with an interaction, it is impossible to determine if the main effect or the interaction is responsible for moving the response. Several eight run factorial designs are shown in Table 11.

Table 11 Eight run factorial experiments.

Experiment	Number of factors	Resolution
2^3	3	infinity
2^{4-1}	4	4
2^{5-2}	5	3
2^{6-3}	6	3
2^{7-4}	7	3

Source: Product work of the author.

One of the issues mentioned earlier is the evaluator or customer's patience when completing a paired-comparison matrix. In an experiment with eight treatments, one is required to solicit 28 paired-comparisons from the customer. This is a large number of comparisons and, in the author's opinion, too many.

Incomplete paired-comparison matrix

Does a method exists that will allow a smaller number of comparisons to be solicited from the customer and still provide sufficient information to complete the paired-comparison matrix? If one assumes a customer's responses will be completely consistent, only $(N-1)$ connected comparisons are required. However, it is not reasonable to assume that a customer will respond with completely consistent responses because some inconsistency is expected. Nevertheless, is it reasonable to assume that one can collect fewer than a full gamut of comparisons, $(N^2-N)/2$, but greater than the minimal number of comparisons, $(N-1)$ connected comparisons, and capture the consistency of the customer's responses? The author will use the preceding conjecture to provide a less taxing and more efficient process to collect customer data while capturing some redundancy to measure the consistency of the judgments.

Producing consistent artificial judgments

If one does not complete the full $(N^2 - N)/2$ comparisons of a paired-comparison matrix, the missing entries must be artificially produced. There are two common methods used for the estimation of unknown comparisons in an incomplete paired-comparison matrix; the Two-stage method and the Harker method (Nishizawa, 2005: 1). This paper will use the Harker method as described by Saaty (Saaty, 2000: 87-88).

To use the Harker method, one completes the entries in the paired-comparison matrix where judgments are available[2]. Where judgments are not available, enter a zero into the corresponding cell of the paired-comparison matrix. Sum the zeroes in each row and add these values to the corresponding diagonal entries, which are always one. After this exercise, all of the cells in the paired-comparison matrix will have an entry. Next, calculate an eigenvector for the matrix and use the eigenvector to supply the missing judgments in the original incomplete matrix. For example, if a judgment for a_{34} is not available, one can obtain the judgment from the eigenvector, $(v_1, v_2, v_3, \ldots, v_n)$, by evaluating $a_{34} = {}^{v_3}\!/_{v_4}$.

Which judgments to solicit

How does one determine the comparisons to solicit from the customer? A simple method to arrive at a minimal set of paired comparisons from a paired comparison matrix is to use the comparisons above the diagonal. An example of the

[2] As previously mentioned, one must provide at least (N-1) connected paired-comparisons.

comparisons above the diagonal in an order eight matrix is shown in Figure 10, where the minimal number of connected paired-comparisons is highlighted.

Figure 10 Minimum set of paired comparisons.

	car 1	car 2	car 3	car 4	car 5	car 6	car 7	car 8
car 1	1	$\mathbf{a_{12}}$	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
car 2	$1/a_{12}$	1	$\mathbf{a_{23}}$	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}
car 3	$1/a_{13}$	$1/a_{23}$	1	$\mathbf{a_{34}}$	a_{35}	a_{36}	a_{37}	a_{38}
car 4	$1/a_{14}$	$1/a_{24}$	$1/a_{34}$	1	$\mathbf{a_{45}}$	a_{46}	a_{47}	a_{48}
car 5	$1/a_{15}$	$1/a_{25}$	$1/a_{35}$	$1/a_{45}$	1	$\mathbf{a_{56}}$	a_{57}	a_{58}
car 6	$1/a_{16}$	$1/a_{26}$	$1/a_{36}$	$1/a_{46}$	$1/a_{56}$	1	$\mathbf{a_{67}}$	a_{68}
car 7	$1/a_{17}$	$1/a_{27}$	$1/a_{37}$	$1/a_{47}$	$1/a_{57}$	$1/a_{67}$	1	$\mathbf{a_{78}}$
car 8	$1/a_{18}$	$1/a_{28}$	$1/a_{38}$	$1/a_{48}$	$1/a_{58}$	$1/a_{68}$	$1/a_{78}$	1

Source: Product work of the author.

Using this method, one would ask all of the customers to evaluate the same seven paired-comparisons. If desired, the evaluation order can be randomized to help average out any potential bias. However, randomization of the question order alone may not rid the question set of bias. Will the importance of the criteria be different if one generated a different minimal set of paired-comparisons for each respondent? The only way to answer this question is with testing. However, one would naturally expect some bias due to the question set. The author believes that generating a minimal set of paired-comparisons randomly for each respondent will average out the bias associated with the order and the content of the questions.

The author proposes a tool that will generate a random set of seven connected pair-comparisons (this number may change depending on the order of matrix) from the original set of 28 possible paired-comparisons. Using this tool will allow one to use less than the maximum, $(N^2 - N)/2$, comparisons, while not introducing bias due to using the same question set for every trial. The development and use of this tool is discussed in Appendices 1 and 2.

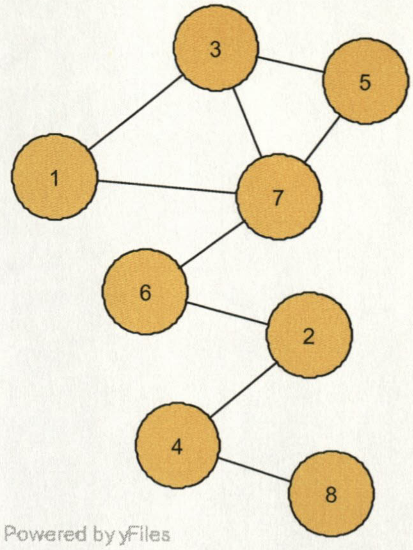
Generating a random consistency index with an incomplete matrix

Using only the minimal number of questions, $(N-1)$, from the full set of possible paired-comparisons, $(N^2 - N)/2$, and artificially generating the remaining comparisons will result in a consistent matrix, however, this is not recommended (Saaty, 2000: 81). One can add additional paired-comparisons, beyond the minimal $(N-1)$ paired-comparisons, to obtain a measure of consistency. As noted previously, Saaty provides a random consistency index for complete paired-comparison matrices up to order 15 (Saaty, 2000: 84). The random consistency index is used as a reference to assess consistency. Saaty recommends that one reconsider judgments when a complete paired-comparison matrix's inconsistency is greater than 10% of the consistency obtained from a random matrix (Saaty, 2000: 85). However, how does one measure the consistency of a paired-comparison matrix when some of the judgments are artificially generated? The Harker and Two-stage methods attempt to supply the most consistent values possible for the missing judgments. Therefore, the resulting matrix is likely more consistent than if the judgments were supplied by the respondent and not artificially generated. Furthermore, the random consistency index for a matrix where all the judgments are created randomly is larger than a random consistency index for a matrix where some of the judgments are created randomly and others are created to be as consistent as possible with the original random judgments.

Therefore, using a random consistency index from a completely randomized matrix to generate the consistency ratio for an incomplete matrix will cause the incomplete matrix to appear more consistent than it really is. To create a consistency ratio for a matrix where not all of the judgments are solicited from the respondent and some of the judgments are created with the goal of consistency with the original judgments, one must generate a unique random consistency index based on the order of the matrix and the number of missing comparisons. Forman has generated random indices for incomplete matrices up to order seven (Forman, 1989).

For instance, assume the nine paired-comparison entries a_{13}, a_{24}, a_{35}, a_{57}, a_{67}, a_{17}, a_{26}, a_{37}, and a_{48} are colleted using a fictitious survey process from a fictitious customer. Also, presume that the nine entries are taken from a possible 28 paired-comparisons (eight alternatives). The graph representing the entries is shown in Figure 11. As one can see and as required, the set contains the minimal seven connected pair-comparisons. However, two additional redundant entries are also collected. The two additional entries will allow one to generate a consistency index.

Figure 11 Graph of random entries.



Source: Product work of the author.

The paired-comparison matrix containing these entries is shown in Table 12.

Each paired-comparison a_{ij}, highlighted in yellow, was collected from the customer.

Each pair-comparison r_{ij}, highlighted in blue, represents a missing judgment.

Table 12 Matrix with random entries.

	car 1	car 2	car 3	car 4	car 5	car 6	car 7	car 8
car 1	1	r_{12}	a_{13}	r_{14}	r_{15}	r_{16}	a_{17}	r_{18}
car 2	$1/r_{12}$	1	r_{23}	a_{24}	r_{25}	a_{26}	r_{27}	r_{28}
car 3	$1/a_{13}$	$1/r_{23}$	1	r_{34}	a_{35}	r_{36}	a_{37}	r_{38}
car 4	$1/r_{14}$	$1/a_{24}$	$1/r_{34}$	1	r_{45}	r_{46}	r_{47}	a_{48}
car 5	$1/r_{15}$	$1/r_{25}$	$1/a_{35}$	$1/r_{45}$	1	r_{56}	a_{57}	r_{58}
car 6	$1/r_{16}$	$1/a_{26}$	$1/r_{36}$	$1/r_{46}$	$1/r_{56}$	1	a_{67}	r_{68}
car 7	$1/a_{17}$	$1/r_{27}$	$1/a_{37}$	$1/r_{47}$	$1/a_{57}$	$1/a_{67}$	1	r_{78}
car 8	$1/r_{18}$	$1/r_{28}$	$1/r_{38}$	$1/a_{48}$	$1/r_{58}$	$1/r_{68}$	$1/r_{78}$	1

Source: Product work of the author.

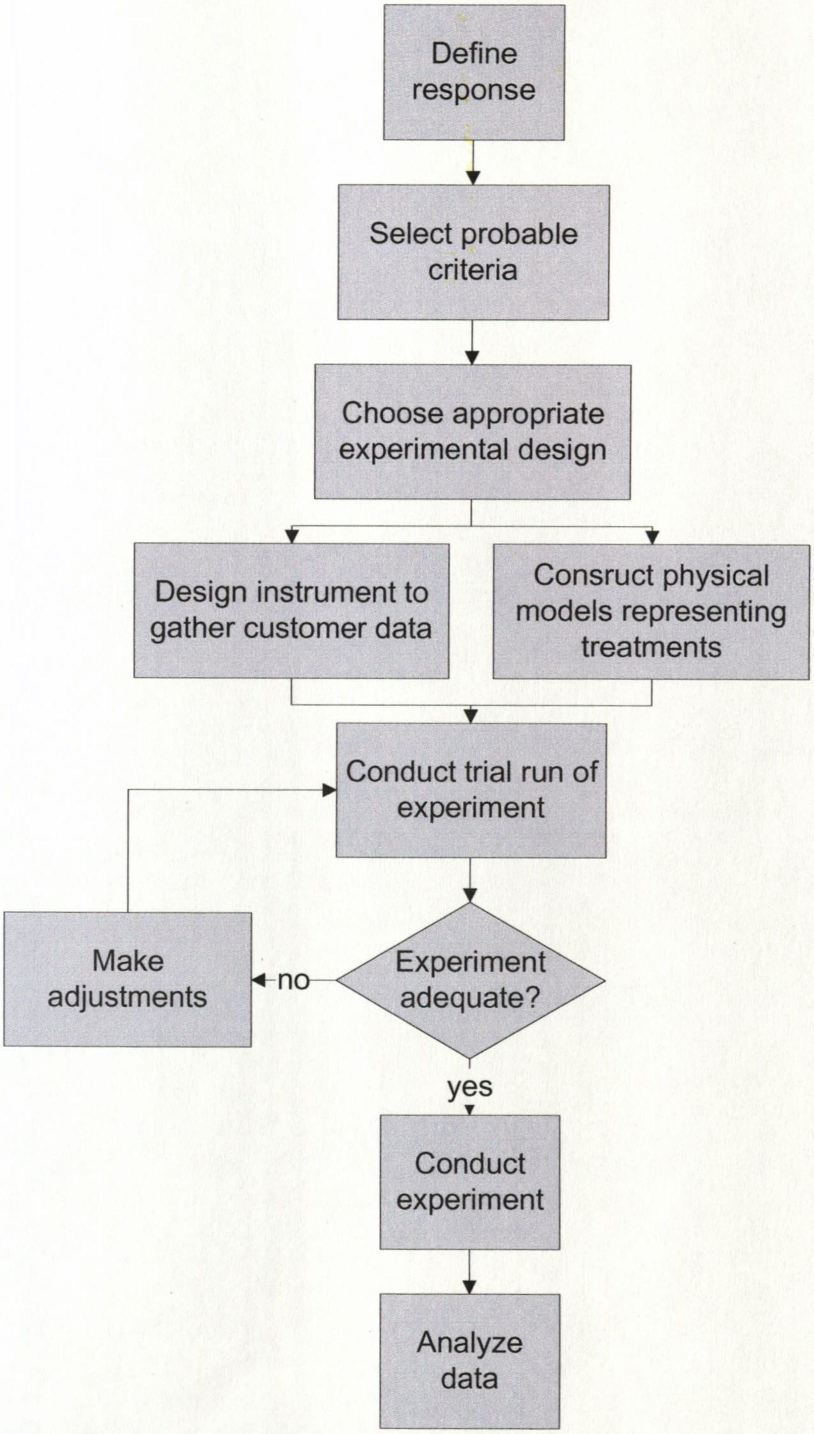
As mentioned previously, to complete the matrix, each missing judgment must be artificially generated such that the greatest consistency is obtained. After the missing judgments are supplied via the Harker method, the matrix is complete and an eigenvector and eigenvalue can be calculated. The eigenvector is used to establish priorities among the alternatives and the eigenvalue is used to calculate a consistency index. However, a random consistency index is required to assess consistency.

To generate the random consistency index, one supplies random values from Saaty's 1-to-9 scale for all the a_{ij}'s. For each set of random a_{ij}, an eigenvalue is calculated and recorded. The eigenvalues are used to generate random consistency indices using the equation $R.I. = (\lambda_{rand\_ave} - n)/(n-1)$ where n is the order of the matrix. After several random consistency indices are recorded, an average random consistency index is calculated and the average random consistency index is divided into the consistency index to create an indicator of consistency. The resulting ratio is used to gauge consistency. The author assumes that the same limits apply. Therefore, an inconsistency of greater than 10% requires a reassessment of the judgments or the homogeneity of the alternatives.

Process summary

A flowchart is provided in Figure 12 that summarizes the proposed process. This section will provide a brief discussion of each process step.

Figure 12 Process flowchart.



Source: Product work of the author.

Define response

To define the response one must ask 'what customer requirement should be maximized, minimized, or optimized to provide the most desirable effect?' The customer requirement can be very nebulous. Requirements such as look nice, feel comfortable and fun to drive are all very real requirements but can be difficult to define.

The response should be worded such that it can be used when soliciting data from the customer. For instance, one might ask the customer to rate two items in reference to how nice they look.

Select probable criteria

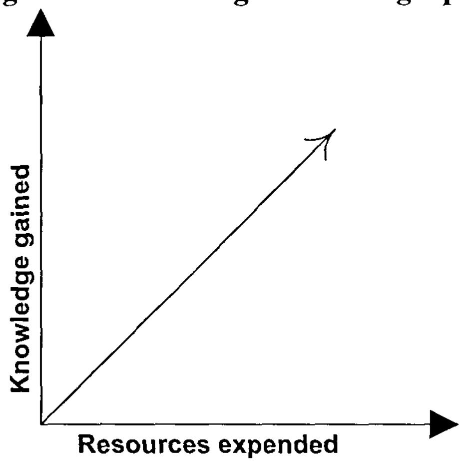
After the response is defined, one must select the probable criteria. One must ask 'what criteria will have an effect on the response?' Only a guess at the criteria is required. If a criterion is not important to the customer, the analysis will show it. The engineer must not only know which criteria are important but he must also know which criteria are not important. If the engineer selects a set of criteria and the response seems to be moved by a criterion or criteria external to the chosen set, the analysis will indicate this. The number of the criteria selected is related to the experiment design.

Chose appropriate experimental design

After the desired criteria set is selected, one must choose an experiment design. The design must be a balance between learning and resources. One should aspire to obtain the most information while using a minimal amount of resources. The graph in

Figure 13 while probably not modeling the resource/ knowledge curve exactly, conveys the general idea that knowledge is not free. One is required to settle on an appropriate level knowledge based on limited resources.

Figure 13 Knowledge/resource graph.



Source: Product work of the author.

The tradeoff of knowledge and resources is mainly determined by the amount of confounding one can tolerate in an experimental design. If one is early in the discovery and only wants to know which factors are important, a low-resolution and highly confounded experiment can probably be tolerated. However, if one is late in the discover process and needs model parameters, a high resolution experiment with little or no confounding is desired.

The available experimental designs are shown in Table 13. As discussed previously, the experiment is restricted to eight treatments, i.e. alternatives.

Table 13 Eight run factorial experiments.

Experiment	Number of factors	Resolution
2^3	3	infinity
2^{4-1}	4	4
2^{5-2}	5	3
2^{6-3}	6	3
2^{7-4}	7	3

Source: Product work of the author.

Design instrument to collect customer data

To collect the data from the customer, one must use some sort of data collection instrument. The author recommends a verbally administered survey

Construct physical models representing treatments

In parallel with designing the data collection instrument, one can construct the physical models that represent the treatments. This can be a very time consuming activity. The models must be constructed such that they accurately represent the product or intended design. However, this can be a challenge when the models are constructed from prototypes.

Conduct trial run of experiment

This step can involve a substantial amount of discovery. The experience of the trial run can bring about modifications to the data collection instrument, the physical models, and even the experiment. The trial run is the engineer's opportunity to optimize the experiment in an effort to control noise and balance resources.

Conduct experiment

Everything up to this point has been in preparation to actually conduct the experiment. During the experiment one must collect the data as called for in the data collection instrument but one must also collect any important observations or comments by the customer. The collection of copious notes documenting external data can be just as important as the collection of the data for the data collection instrument.

Analyze data

Several methods can be used to analyze the data. One can start with graphical techniques to analyze the data and proceed to more complicated statistical methods to uncover less obvious information. However, one should always look at the data from a practical viewpoint and ask "why?"

Refrigerator Dispenser Cavity Lighting Experiment

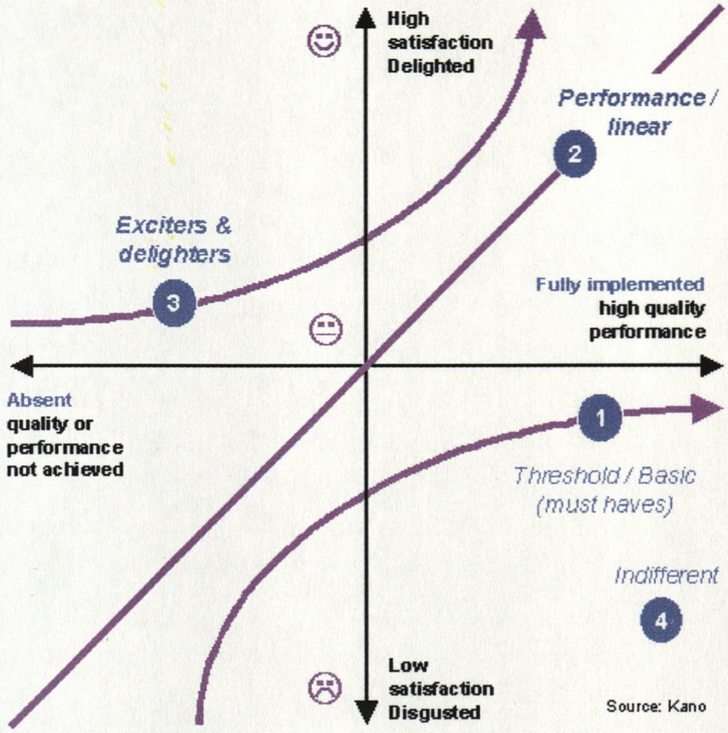
Introduction

How effective is the proposed experimental process at translating customer requirements into technical targets? The best way to validate the proposed process is to trial it in a real scenario. To demonstrate the process, a product feature is required with a definition that is highly dependent upon subjective customer input. Furthermore, it is desirable that such a feature be innovative while adding value.

The author selected refrigerator dispenser-cavity lighting to test the proposed process. This product feature was selected because it is very dependent on subjective customer input for its definition. Additionally, due to the potential aesthetic impact of this feature on the retail sales floor, it can add a great deal of perceived value to the product.

Like many product features, refrigerator dispenser lighting follows the Kano model. The Kano model, as shown in Figure 14, mainly states that in a competitive market, a new and exciting product feature over time will lose its impact on the customer and will become an expected feature that is required to maintain customers, but no longer has the capacity to draw new customers. The loss of excitement about a product feature is the result of equivalent or better offerings by the competition and a subsequent loss of product differentiation. In a highly competitive environment, exciting new features provide the differentiation that is required to increase market share and keep existing customers from migrating to competitive products. Therefore, understanding a customer's needs, even before they do, will provide a competitive edge.

Figure 14 The Kano model.

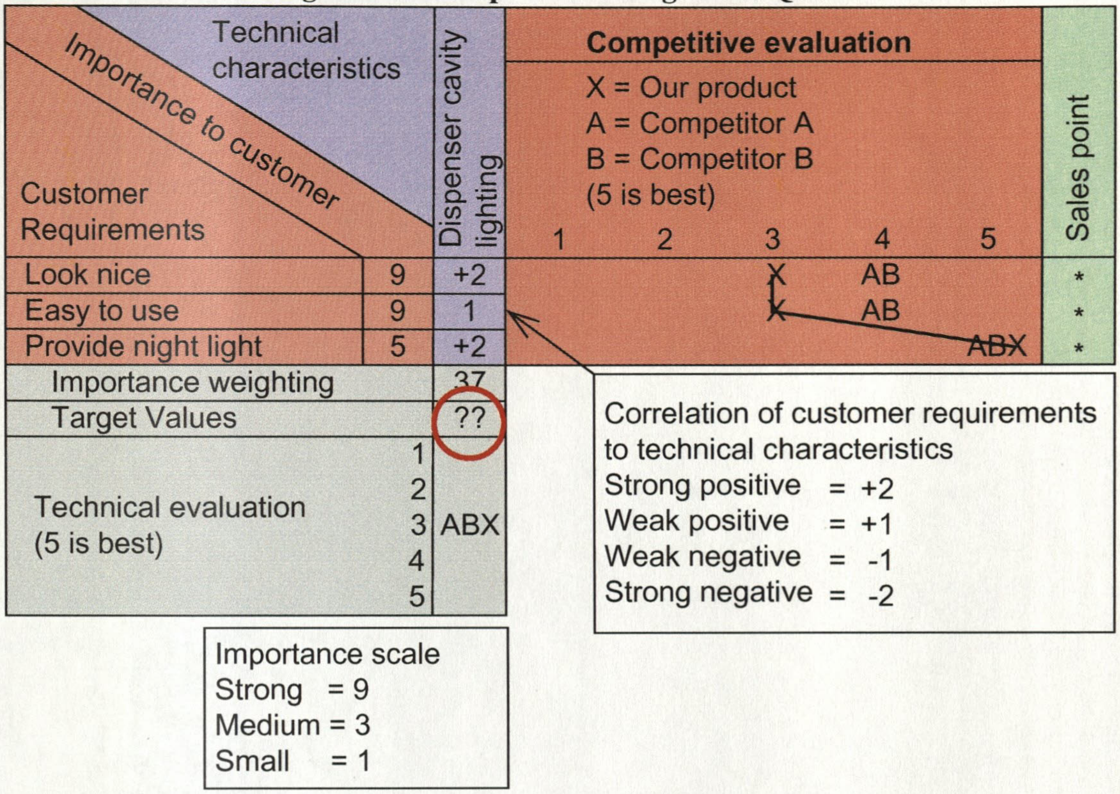


Source: http://www.betterproductdesign.net/tools/definition/kano.htm.

Low cost white light emitting diodes (LEDs) have provided an opportunity to shift dispenser cavity lighting from the 'threshold/ basic' category to the 'exciter & delighter' category. This feature is reasonably easy to implement. However, if not implemented effectively, it will provide little competitive advantage. Several competitive product offerings with this feature currently exist. However, a competitive advantage may be available to the manufacturer who better understands the customer.

Dispenser cavity lighting seems to serve three purposes. It helps the user to see what they are doing, it is used to enhance the appearance of the product, and it is used as a night light for the kitchen. A QFD table might show these functions as the 'product must look nice', 'product must be easy to use' and 'product must provide a light in the dispenser area'. If one were to include these in a refrigerator Quality Function Deployment (QFD) matrix, the matrix might appear as in Figure 15. Figure 15 represents the author's evaluation of the feature dispenser cavity lighting in a QFD matrix. No real customer data was gathered to assemble the matrix however, the author feels the results are close to information an actual customer might provide. The product evaluated in this QFD matrix is what the author had available for experimentation and the two competing products in the QFD matrix are highly featured competitive products. One can immediately surmise that the technical characteristic, dispenser cavity lighting, has a high correlation with several important customer requirements. The 'importance weighting' has no meaning by itself however, with the high customer importance and high correlation, one would expect that this feature would be high on the list of items competing for resources. The target value, circled in red in Figure 15, is shown as unknown. The target for this feature is not a case of maximization or minimization but of optimization based on customer input. The QFD exercise provides additional evidence that dispenser lighting is a good example to use for validation of the proposed experimental process.

Figure 15 Excerpt from refrigerator QFD.

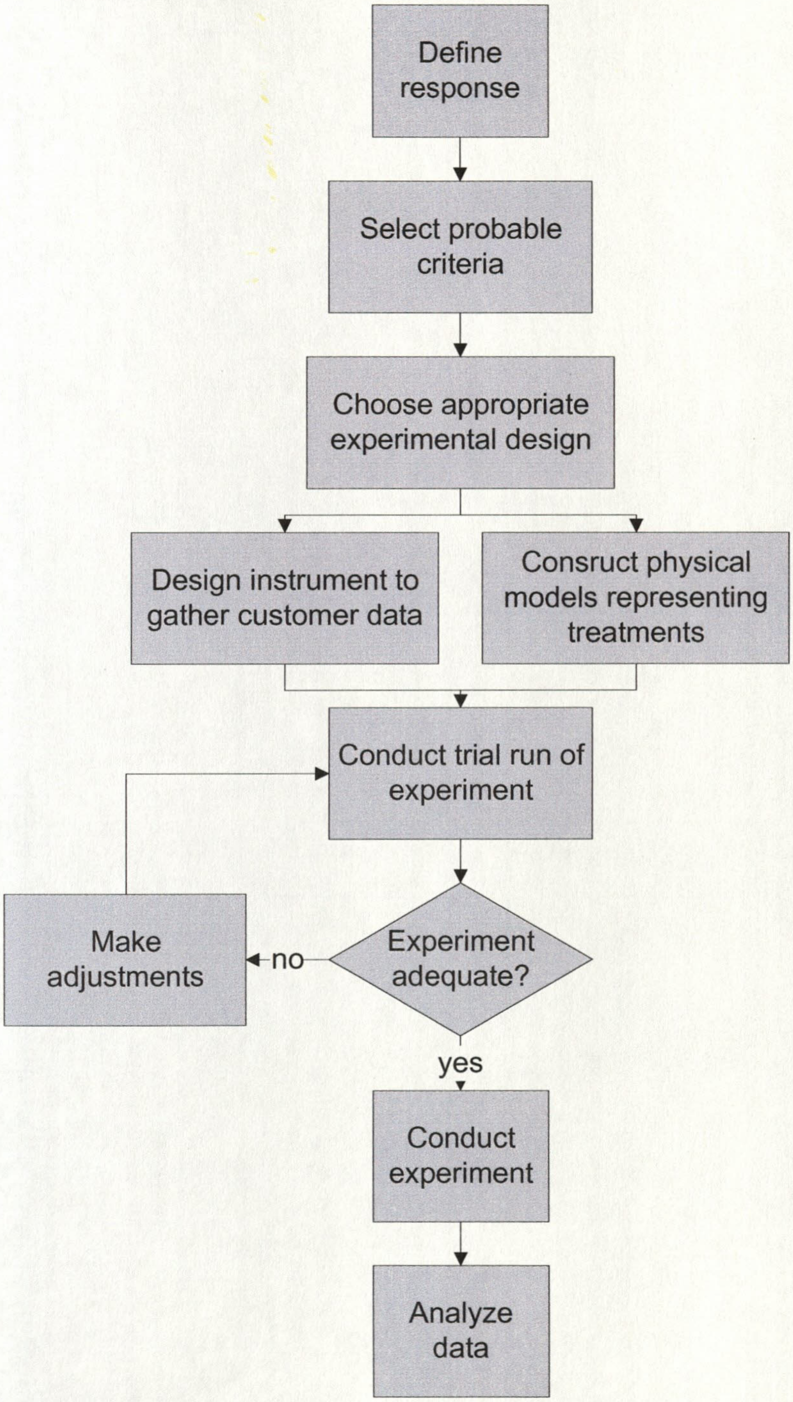


Source: Product work of the author.

Additional evidence to support the deployment of resources to develop dispenser cavity lighting exists in the analogy between dispenser cavity lighting and retail lighting. Retail lighting is specifically designed to enhance a products appearance and catch the customer's eye. According to the Retail Lighting Guide produced by designlights.org, "The selection of the right lighting can be a major contribution to retail sales. Lighting can establish a store's image, lead customers inside, focus their attention, make the products attractive and visible, and in general encourage purchasing" (Small Retail Lighting Knowhow). Furthermore, "90% of purchasing decisions are made at the point of sale" (lighting design lab, 1). Retailers devote a lot of resources to design lighting with the intent of attracting customers and selling products. Based on the importance placed on retail lighting, one can easily form the conclusion that dispenser cavity lighting may possess the same potential to attract customers.

The next section will explain the process by following the flowchart shown in Figure 12. Figure 12 is reproduced for convenience below as Figure 16. Based on the QFD given in Figure 15, the response chosen was 'looks nice' and was chosen to support the QFD. The author then selected a set of probable factors that may determine how nice the dispenser cavity lighting looks.

Figure 16 Process flowchart.



Source: Product work of the author.

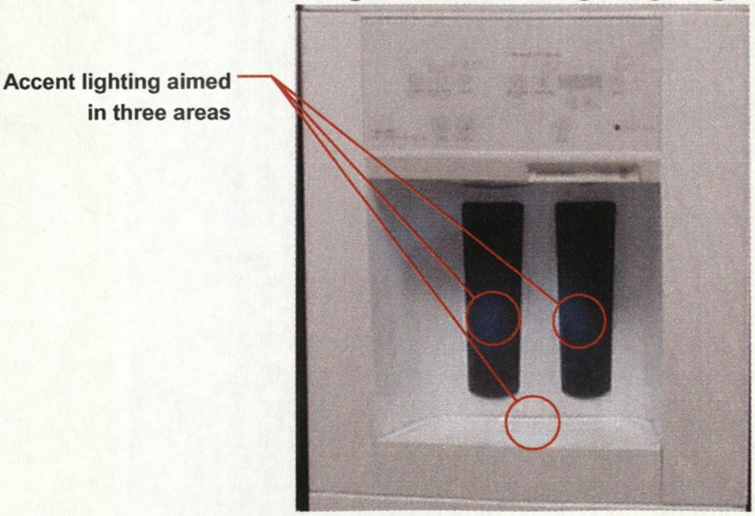
Based on the author's experience and research, four factors were selected. These factors are listed in Table 14. The factors were implemented using modules, which contain multiple LEDs. The color of the LED modules was dependent upon the color of the LEDs assembled into the module. The intensity of the LED modules was dependent upon the quantity of the LEDs in the module and the amount of electrical current passing through the LEDs. The background lighting module was configured to provide non-directional lighting to flood the dispenser cavity. The accent or target lighting module was configured to provide three cones of light to specific areas in the dispenser cavity as shown in Figure 17.

Table 14 Lighting factors.

Background lighting color
Accent lighting color
Background lighting intensity
Accent lighting intensity

Source: Product work of the author.

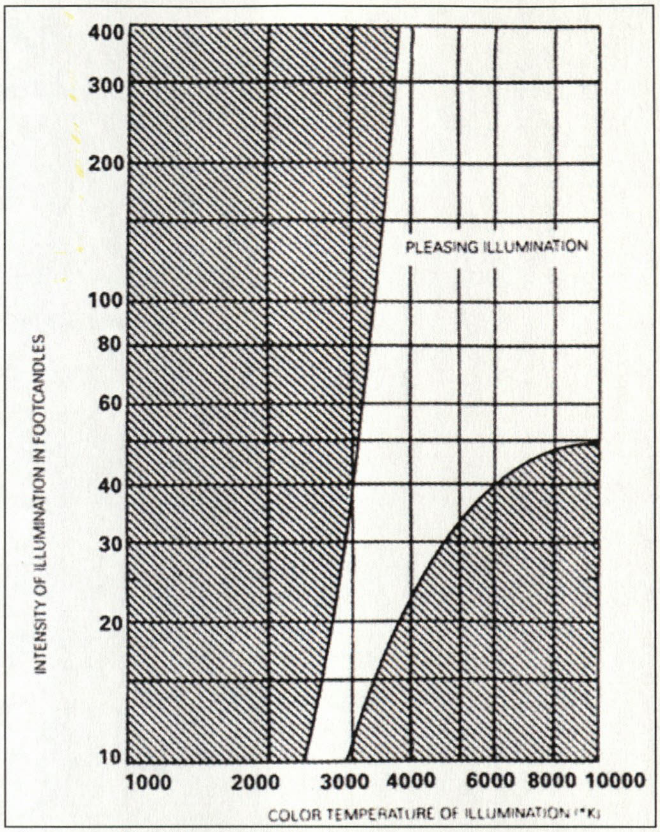
Figure 17 Accent lighting targets.



Source: Product work of the author.

Justification for the selection of color and lighting intensity as factors are rooted in the fact that color and lighting are closely related and people are known to have preferences regarding these phenomena. For example, A. A. Kruithof published a chart in 1941 (see Figure 18), which illustrates the preferences of individuals regarding intensity, color temperature, and the 'pleasant' quality of a light source (Kruithof, 1941: 65-96). Kruithof's findings indicate that one's color preference will change in relation to the intensity of the light source. Evidence also exists that color preference is related not only to the lighting intensity but also to the environment that it is located in Birren states that "no list of color associations is adequate unless in takes into consideration these subjective as well as objective aspects. For reactions will differ as a person associates color with the outside world or with himself" (Birren, 1961: 142). In summary, one cannot conclude that a certain lighting intensity and color is always preferred based on past situations and one should consider each scenario within the environment it is to be used in.

Figure 18 Kruithof curve.



Source: Kruithof, 1941: 65-96.

The author then selected the experimental design. As noted earlier, one can choose from five factorial designs that each contains eight treatments. Four factors were selected as potentially important to the customer for determining how nice dispenser lighting looks. Therefore, a 2^{4-1} factorial design was selected. Additionally, the main effects in a 2^{4-1} factorial experiment are not confounded with two-factor interactions. The author felt that two-factor interactions might be important in this experiment. Therefore, a minimal amount of confounding was desired. The next larger experimental design, a 2^{5-2} factorial design, which allows an additional factor to be included, was not selected due to the lower resolution and the lack of a requirement for an additional factor. The selected design, a 2^{4-1} design, is a resolution four experiment, which only confounds main effects with three way interactions and two-way interactions with two-way interactions. The experimental design selected is highlighted in Table 15.

Table 15 Eight run factorial experiments.

Experiment	Number of factors	Resolution
2^3	3	infinity
2^{4-1}	4	4
2^{5-2}	5	3
2^{6-3}	6	3
2^{7-4}	7	3

Source: Product work of the author.

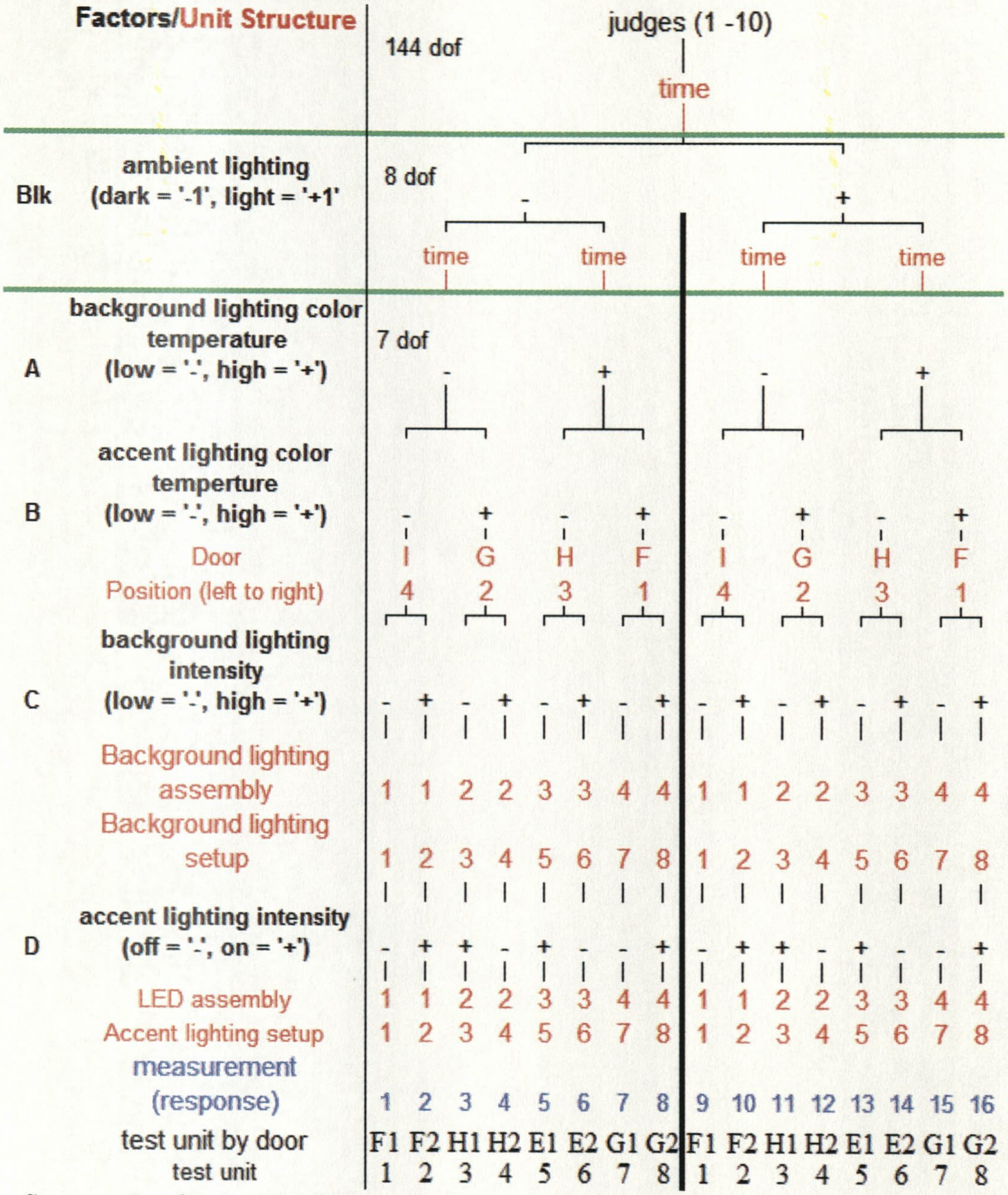
The selected experimental design requires eight treatments. However, upon investigation, the construction of eight individual treatments required a significant amount of resources and due to the size of a refrigerator door, also required a significant amount of space. Therefore, the author decided to vary the intensity of the LED modules with like colors by adjusting the electrical current and not by switching between modules. This modification to the experiment resulted in a substantial resource savings by allowing the eight treatments to be represented with the construction of only four physical models and some additional wiring. However, the tradeoff required that relative comparisons of some treatments would take place on the same door. The consequence of this was that treatments compared on the same door would be switched on and off instead of being statically displayed on two different doors. As a result, the respondent was required to store a mental image, albeit very briefly, of the treatment not currently shown.

To increase the inference space of the experiment, each respondent was asked to complete the survey under ambient lighting conditions similar to a retail environment and additionally, in total darkness. Due to the inclusion of two different lighting environments, information was gathered regarding the technical targets related to the customer requirements of "look nice' and "provide night light". This experiment did not directly address the customer requirement of 'easy to use', however the low correlation with lighting indicates it can probably be included in another experiment directed at use and not aesthetics.

In addition to the factors in the experiment, there were also other uncontrolled and/or unknown factors. One must attempt to document and sometimes understand these rogue factors. These factors constitute the noise structure of an experiment and can help explain departures from the resulting model. If a noise factor is thought to be significant, it can be included as a controlled factor in future experiments and therefore, become part of the model.

The sampling structure for the experiment is shown as a factor relationship diagram (FRD) in Figure 19. An FRD displays the actual sampling structure of the experiment plus several other additional details specifying the experiment (Bergerud). The black items indicate the controlled factors in the experiment and red items indicate the noise structure of the experiment. The green line at the top of the FRD indicates a line of restriction where randomization does not occur from one side of the line to the next. The blue items at the bottom indicate the actual measurements and the text at the bottom of the diagram is some additional information documenting test units. The degrees of freedom (dof) associated with level in the FRD are shown in Figure 20.

Figure 19 Factor relationship diagram.



Source: Product work of the author.

Figure 20 Degrees of freedom.

144 dof	j ... j^9, (j ... j9)*(Blk, Blk*A, Blk*B, Blk*C, Blk*D, Blk*AB, Blk*AC, Blk*AD), (j ... j9)*(A, B, C, D, AB, AC, AD
8 dof	Blk, Blk*A, Blk*B, Blk*C, Blk*D, Blk*AB, Blk*AC, Blk*AD
7 dof	A, B, C, D, AB, AC, AD

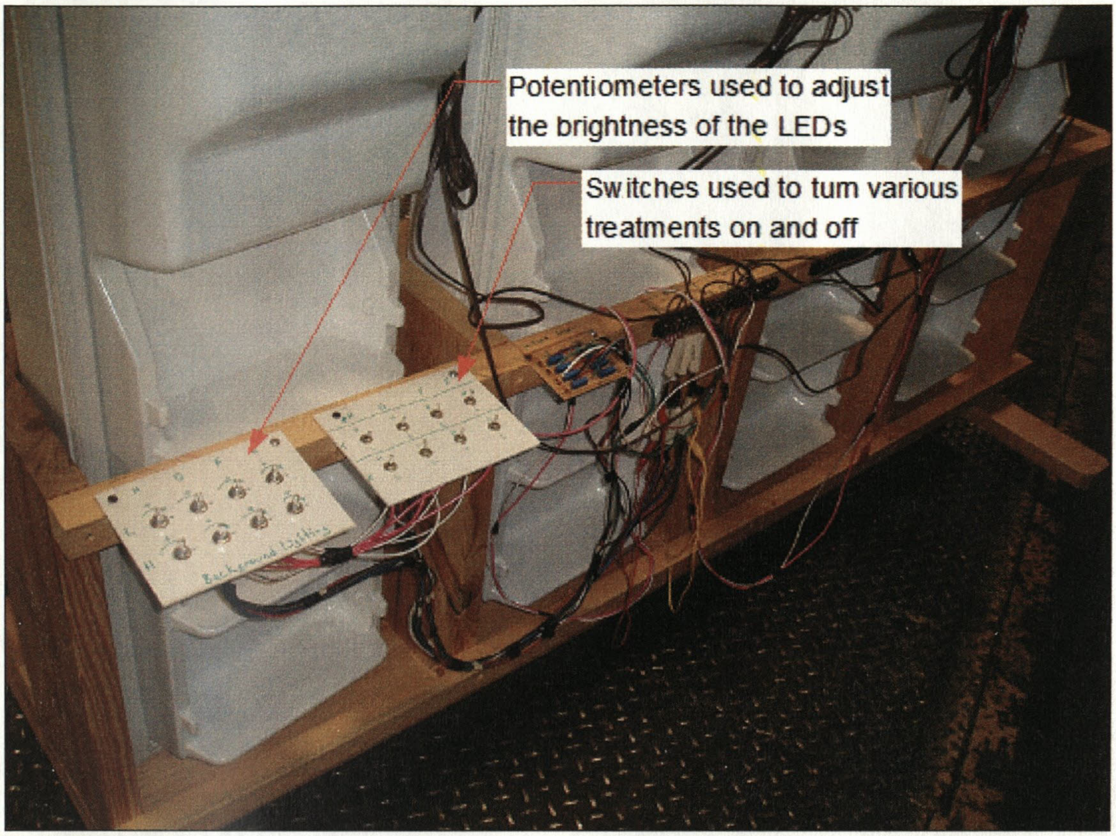
Source: Product work of the author.

The author next constructed the physical models and designed the data collection instrument. The four physical models representing the eight treatments are shown in Figure 21. As one can see, the doors are labeled as ‘E’, ‘F’, ‘G’, and ‘H’. This labeling structure was an attempt to avoid any potential bias associated with the labeling scheme of ‘A’, ‘B’, ‘C’ and ‘D’ that is commonly used as a grading system in education. If this bias exists, a respondent may automatically give preference to a product labeled ‘A’. The fixture controls that adjust the LED module brightness and their on/off status are shown in Figure 22. The electrical schematic of the fixture controls is shown in Appendix 3.

Figure 21 Four physical models of eight treatments.

Source: Product work of the author.

Figure 22 Fixture controls to switch between treatments.



Source: Product work of the author.

In parallel to the fabrication of the physical models, the author designed the data collection instrument or the survey. Put very simply, “A survey is a systematic method of collecting information from a selected group of people by asking a series of questions” (Houston, 2). The author originally trialed several different written surveys, however, the surveys were awkward and distracting to the respondents. Therefore, the author decided to administer the survey verbally. The verbally administered survey was designed using good survey design practices such as those given in Houston and E. Zimmerman. The survey instructions, administered questions, rating scale handout, and data collection form/ survey sample can be found in Appendix 4. As previously stated, the entire survey was administered verbally. However, the respondent was given a handout, which contained the rating scale. The respondents used the handout as a reference during the verbal administered survey.

The author then validated the experimental design by performing trial runs or conducting pretests. As mentioned previously, the survey was refined by conducting several pretests. Additionally, trial runs played a significant role in the refinement of the factor levels. Originally, two brightness levels were selected for the target lighting module, but it was found that insufficient discrimination existed between the two levels. Therefore, the lower level setting was configured as no target lighting or zero brightness and the high-level setting was configured to provide the maximum output available from the target lighting module. A similar problem did not exist for the background lighting modules, due to the higher available lighting intensity. The high level setting of the background lighting module was set near the maximum output available and the low level was set noticeably different but at a level to provide enough light to flood the dispenser cavity.

Due to the factor 'accent lighting intensity' having a low setting of off, the factor 'accent lighting color' is 'watered down' because only one-half of the runs in the experiment are done with the target lighting on. To get a better understanding of the accent lighting color, this factor was analyzed separately by only examining the trials where the accent lighting was on. The author also viewed the factor 'accent lighting intensity' with caution due to the high level setting containing two levels of accent lighting color while the low level setting of accent lighting intensity possessed no accent lighting color.

The characteristics for the LEDs selected for the experiment are listed in Table 16. LED1 and LED2 were used for the target lighting. Each accent lighting module contains three LED1s or LED2s. These LEDs are small low power LEDs and do not produce a large amount of light. LED3 and LED4 were used for the background lighting. These LEDs are much brighter (and more costly) than the LEDs used in the target lighting modules (LED1 and LED2). Consequently, they also require a considerable amount of power compared to the smaller LED1 and LED2. Figure 23 displays the various LEDs represented on a CIE 1931 chromacity diagram. The CIE 1931 chromacity diagram is a tool used to specify colors by means of coordinates. One can discern that when LED1 and LED4 are used in the same dispenser cavity, LED1 will probably not be noticeable due to the much higher intensity of LED4. However, when LED2 is paired with LED3, one should detect a very noticeable difference.

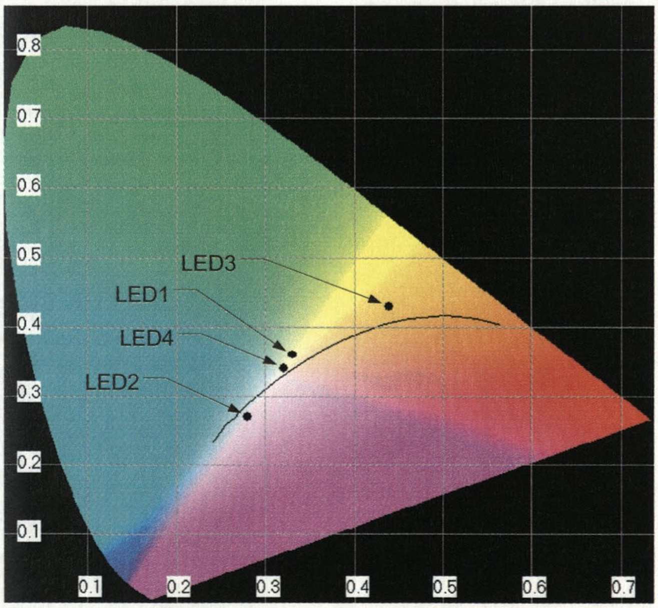
Correlated color temperature (CCT) is also given in Table 16 and represented in Figure 23 by the curve running across the diagram. The correlated color temperature of an object is the temperature at which the color of a black body most closely matches the perceived color of the object. Light sources that lie on or near the black body curve look more natural than light sources that lie far away from the black body curve. Therefore, light sources that are used in situations where aesthetics are important lie on or very close to the black body curve. An example of a light source that does not lie near the black body curve and appears unnatural is a Low Pressure Sodium (LPS) lamp. These lamps are common fixtures in outdoor lights around roads and industrial sites. Objects viewed under these lamps will have poor color rendering and the objects will always appear to be the color of the LPS lamp. Consequently, these light sources are not used in situations where good color rendering is desired. However, in situations where color rendering is not an issue, their energy efficiency makes them very popular.

Table 16 LED characteristics.

LED	Description	Correlated Color Temperature	Typical Luminous flux (lm)	Approximate color coordinates	
				x	y
LED1	target lighting	5600	*1.34	0.33	0.36
LED2	target lighting	11200	*1.34	0.28	0.27
LED3	background lighting	3150	20.0	0.44	0.43
LED4	background lighting	6300	40.5	0.32	0.34
* represents an approximate conversion from mcd to lm					

Source: Product work of the author.

Figure 23 LEDs represented on CIE 1931 chromacity diagram.



Source: Product work of the author.

Results

The experiment was completed using 10 respondents. The only demographic information collected was gender and no attempt was made to randomize or evenly distribute gender across the sample space. However, in practice, the collection of any additional information is always a good idea, especially demographic information. This information can be analyzed along with the controlled factors. For example, one might find that a particular feature is very popular among women from age 25 to 40 but is not popular among any men. This information will allow marketing to target the segment of the population that favors this feature. The gender data is shown in Table 17.

Table 17 Gender of sample space.

Run Number	Gender
1-2	male
3-4	female
5-6	male
7-8	female
9-10	male
11-12	male
13-14	male
15-16	female
17-18	male
19-20	female

Source: Product work of the author.

The respondents were informed that any additional comments would be noted during the survey. The comments are sometimes more revealing than the data. The comments give insight into why someone may or may not like a particular treatment.

The author collected the comments while conducting the survey. More information would have been captured if an extra resource, i.e. person, were included to collect comments. Conducting the survey and collecting comments is too much for one person to do effectively. Comments were collected for both the light and the dark parts of the experiment. The comments are listed in Table 18.

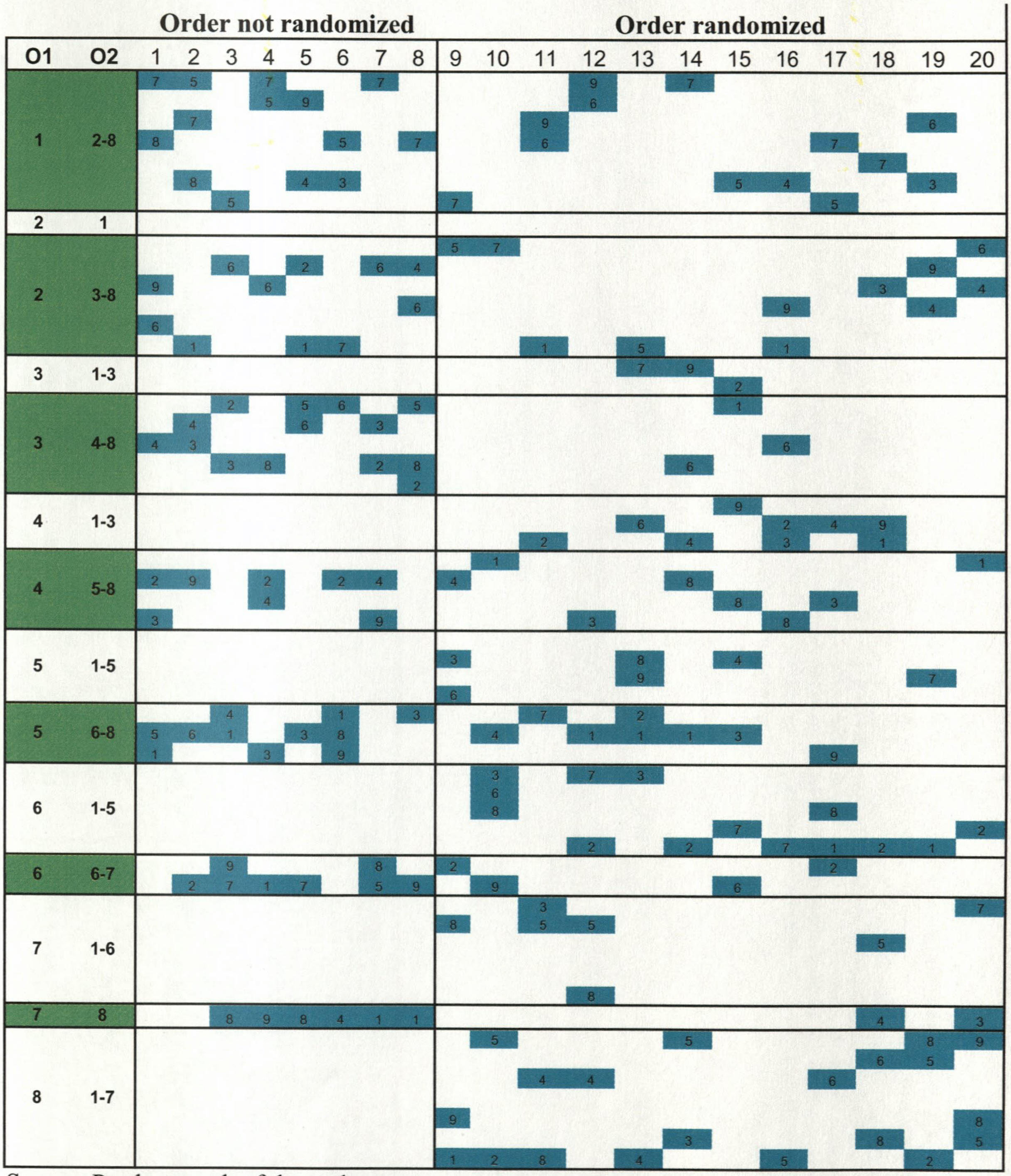
Table 18 Collected comments.

Run Number	Comments
1a	none
1b	none
2a	none
2b	none
3a	none
3b	none
4a	Did not light spots (accent lighting)
4b	Did not like dots (accent lighting)
5a	none
5b	none
6a	none
6b	none
7a	none
7b	none
8a	T1; like the blue, T2; F too bright, T4; F too yellow, T6; likes blue halo
8b	Blue during day doesn't look good but is T5; better at night, blue is distraction with yellow
9a	none
9b	Doesn't like paddle lights
10a	T4; Doesn't like the yellow, T8; Doesn't like the yellow
10b	none

Source: Product work of the author.

As the FRD indicates, within the light and dark blocks of the experiment, the data were completely randomized to average out any bias related to question order. However, the author overlooked the order with respect to which option was presented to the respondent first. Questions one through eight always have the lowest numbered option presented first. This source of potential bias was corrected in trials nine through twenty. One can easily see the lack of randomization in Table 19. In Table 19, the 'O1' column represents the first option and the 'O2' column represents the second option. All of the completed data collection forms can be found in Appendix 6. Screenshots of the Excel spreadsheet used to generate the comparison schedule, i.e. question set, can be found in Appendix 5.

Table 19 Order of presented options.



Source: Product work of the author.

The compiled data are shown in Tables 20 through 22. Tables 20 and 21 contain the priority vectors from each trial and Table 22 displays the consistency ratios for each run. One must remember that the ratings are normalized. Therefore, no single treatment can exceed one. Practically, a rating of 0.5 or higher is unusual. Figure 24 is a histogram of the ratio data gathered in this experiment. One can see that only two treatments exceeded 0.5. This is an important concept when one is accustomed to analyzing experiments where the data are not normalized. When the data are not normalized, it is common for treatments to produce responses greater than average. Screenshots of the Excel spreadsheets used to calculate the priority vectors can be found in Appendix 5.

There are many ways to analyze data and arrive at the same conclusions. The author performed some simple practical and graphical analysis followed by a normal plot and pareto plot of the estimated factor effects.

Table 20 Tests 1 - 10 data.

test		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ambient		**light**	**dark**	**dark**	**light**	**light**	**dark**	**dark**	**light**	**light**	**dark**
gender		**m**	**m**	**f**	**f**	**m**	**m**	**f**	**f**	**m**	**m**
Alternative (Treatment)	**1**	0.023	0.020	0.028	0.031	0.008	0.042	0.003	0.075	0.043	0.030
	2	0.057	0.076	0.138	0.065	0.081	0.021	0.017	0.201	0.174	0.027
	3	0.018	0.049	0.183	0.015	0.027	0.005	0.036	0.037	0.170	0.027
	4	0.041	0.012	0.051	0.049	0.104	0.034	0.107	0.402	0.011	0.032
	5	0.222	0.224	0.043	0.230	0.104	0.091	0.096	0.027	0.044	0.256
	6	0.124	0.093	0.323	0.345	0.140	0.267	0.467	0.098	0.116	0.151
	7	0.150	0.067	0.078	0.103	0.082	0.405	0.223	0.129	0.047	0.046
	8	0.366	0.459	0.155	0.163	0.454	0.137	0.052	0.032	0.396	0.431

Source: Product work of the author.

Table 21 Tests 11 - 20 data

test		11	12	13	14	15	16	17	18	19	20
ambient		**light**	**dark**	**light**	**dark**	**dark**	**light**	**light**	**dark**	**light**	**dark**
gender		**m**	**m**	**m**	**m**	**f**	**f**	**m**	**m**	**f**	**f**
Alternative (Treatment)	**1**	0.013	0.027	0.058	0.012	0.006	0.063	0.026	0.051	0.041	0.059
	2	0.055	0.056	0.017	0.037	0.030	0.039	0.001	0.013	0.027	0.017
	3	0.034	0.058	0.084	0.036	0.139	0.019	0.043	0.025	0.066	0.044
	4	0.082	0.054	0.004	0.101	0.016	0.089	0.009	0.074	0.019	0.021
	5	0.061	0.049	0.154	0.065	0.252	0.437	0.175	0.076	0.382	0.108
	6	0.412	0.326	0.563	0.470	0.091	0.155	0.585	0.314	0.098	0.118
	7	0.053	0.120	0.019	0.187	0.066	0.022	0.040	0.372	0.132	0.484
	8	0.291	0.312	0.102	0.091	0.401	0.176	0.122	0.075	0.234	0.149

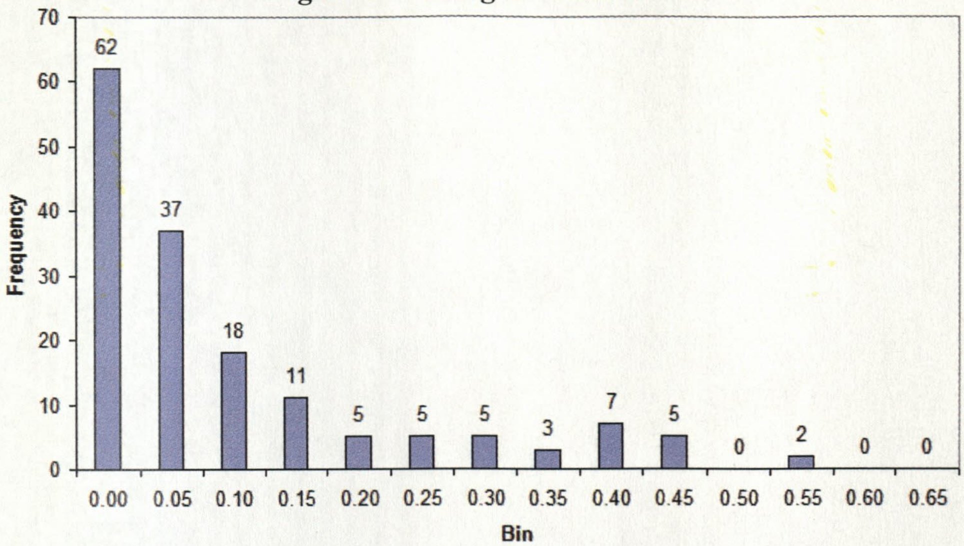
Source: Product work of the author.

Table 22 Consistency Ratios.

Run	CI	n	m	CR	ambient	gender
1	0.0544	8	19	0.2367	light	male
2	0.0242	8	19	0.1056	dark	male
3	0.0151	8	20	0.1539	dark	female
4	0.0175	8	20	0.1779	light	female
5	0.0112	8	19	0.0488	light	male
6	0.0473	8	19	0.2062	dark	male
7	0.0179	8	19	0.0782	dark	female
8	0.0184	8	19	0.0804	light	female
9	0.0039	8	19	0.0168	light	male
10	0.0193	8	19	0.0841	dark	male
11	0.0045	8	19	0.0197	light	male
12	0.0226	8	19	0.0985	dark	male
13	0.0606	8	19	0.2640	light	male
14	0.0003	8	19	0.0014	dark	male
15	0.0192	8	19	0.0839	dark	female
16	0.0093	8	19	0.0404	light	female
17	0.0417	8	19	0.1818	light	male
18	0.0316	8	19	0.1377	dark	male
19	0.0219	8	19	0.0956	light	female
20	0.0209	8	19	0.0913	dark	female

Source: Product work of the author.

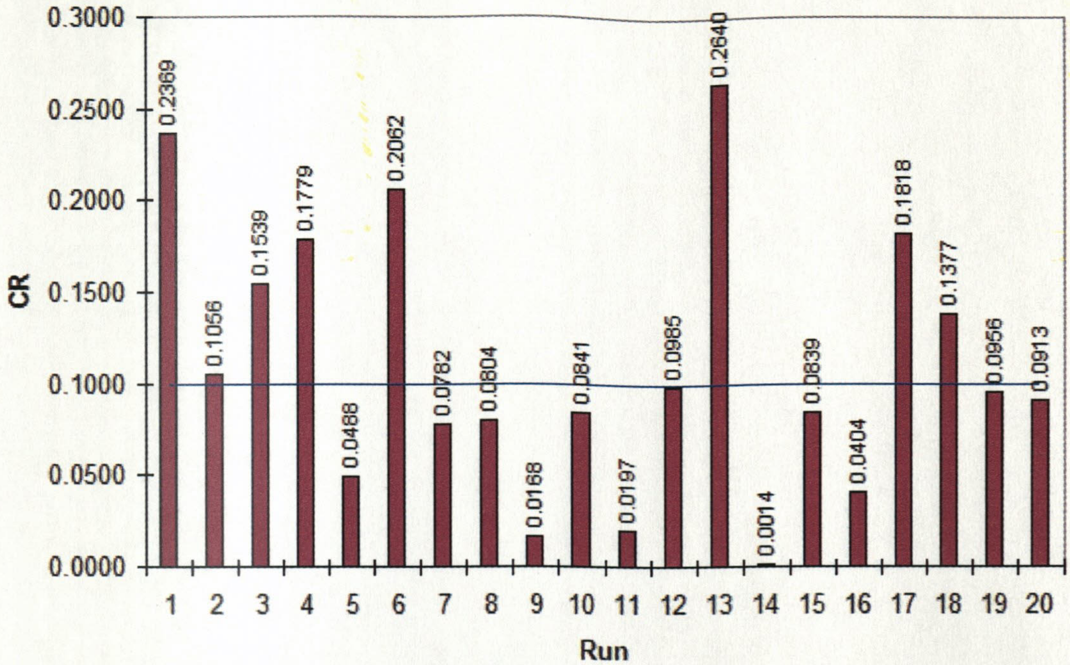
Figure 24 Histogram of ratios.



Source: Product work of the author.

When collecting data via relative comparisons, consistency is important. If the respondent is not consistent enough in their judgments, one should suspect a problem with the experiment or with the respondent's values. However, one must remember that some inconsistency is expected and normal as the respondent refines their internal standard during the process. The consistency data collected for the dispenser cavity lighting experiment is shown below in Figure 25.

Figure 25 Consistency ratios.



Source: Product work of the author.

As noted previously, Saaty recommends a consistency ratio of 0.10 or less (Saaty, 2000: 84). However, random indices are not available for incomplete matrices of order eight. Forman generated random indices for incomplete matrices up to and including order seven (Forman, 1990). Using similar methods, the author generated two random indices for an order eight matrix. The first random index is for 20 missing comparisons and the second random index is for 19 missing comparisons. These random indices should provide a reliable reference for consistency ratios. However, the author's methods were much less extensive and less precise than the methods used by Foreman. Therefore, these random indices should only be used in absence of established random indices.

To verify the methods used to calculate these indices, the author generated a consistency index for a random matrix where n = 7, m = 12, to compare with Forman's values of the same type matrix where n is the order of the matrix and m is the number of missing comparisons. The author's values are shown in Table 23 and Forman's values are shown in Table 24. The author obtained a value of 0.310 (rounded) while Forman obtained a value of 0.32232. Additionally, one can gain some confidence in the author's numbers by comparing the difference between random indices as the number of missing comparisons increases. For example, one can see that the 'IC Avg' for n = 7, m = 13 (redundancy = 2) is nearly double the value for n = 7, m = 14 (redundancy = 1) in Forman's table. The author's value for n = 8, m = 19 (redundancy = 2) is also approximately twice the author's value of n = 8, m = 20 (redundancy = 1).

Table 23 Author's random consistency indices.

Trials	n	m	Redundancy	IC Avg	IC Std Dev	Standard Error
1000	8	19	2	0.229	0.224	0.007
500	8	20	1	0.098	0.148	0.007
3640	7	12	3	0.310	0.255	0.005

Source: Product work of the author

Table 24 Forman's random indices.

Trials	n	m	Redundancy	IC Avg	IC Std Dev	Standard Error
13471	7	10	5	0.55072	0.29016	0.0025
12482	7	11	4	0.41665	0.27930	0.0025
10969	7	12	3	0.32232	0.26182	0.0025
8760	7	13	2	0.22515	0.23398	0.0025
5452	7	14	1	0.11962	0.18459	0.0025

Source: Forman, 1990.

Referring back to Figure 25, twelve of the trials resulted in consistency ratios of less than 0.10, five of the trials resulted in consistency ratios greater than 0.10 and less than 0.20, and three of the trials resulted in consistency ratios of greater than 0.20. Is this too much inconsistency? The author feels that this level of consistency while not ideal is still not of great concern and any major factors should still be evident in the analysis. Additionally, consistency may be an indicator that the experiment needs improvement or the factors are ambiguous. Therefore, one should look at the consistency index as a tool to use for more than indicating a problem in a respondent's judgments.

In an effort to understand the source of the inconsistency in this experiment, the author interviewed several respondents concerning their experience. The author found that some respondents were developing their reference standard as the experiment progressed. Nonetheless, some development of one's standard is expected. However, based on the feedback from the respondents, the author felt the adjustments during the experiment could have been a source of more inconsistency than expected. For example, one respondent indicated that midway through the experiment their preference for the 'spots' changed.

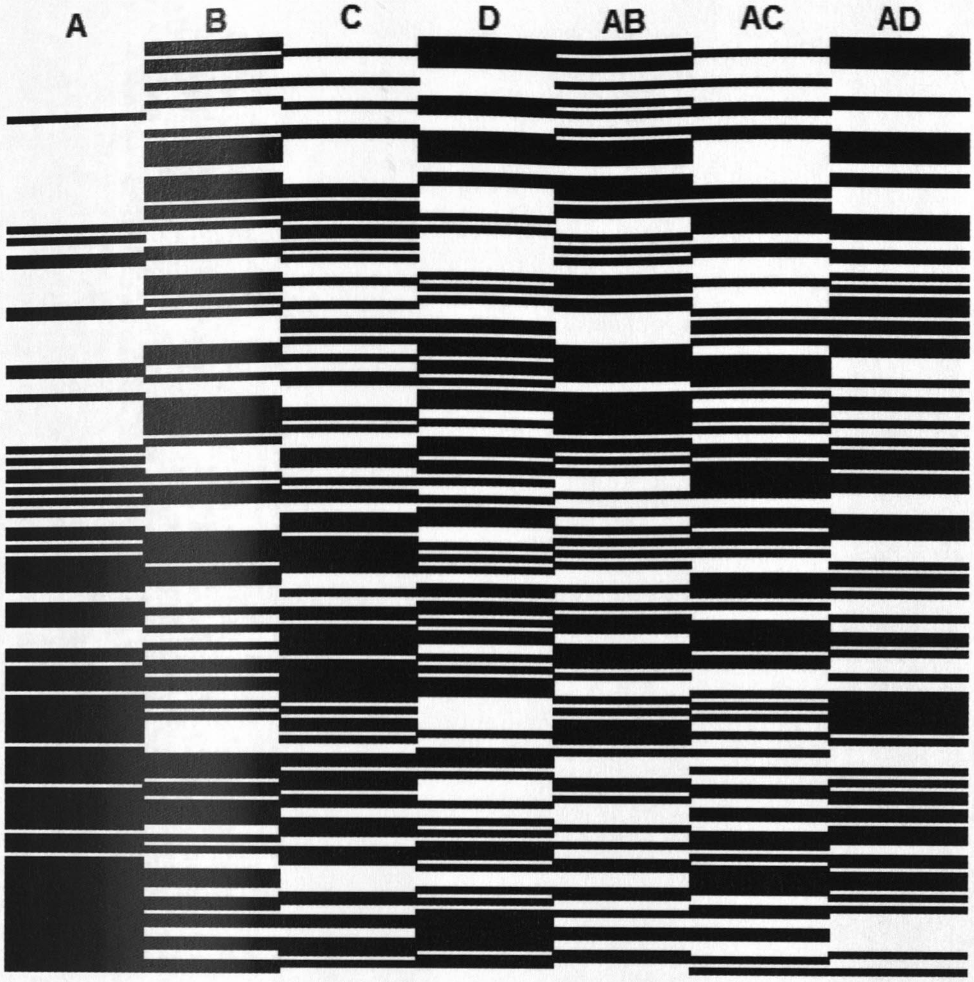
Would a preview of the treatments have helped the respondents establish a standard before the start of testing, therefore improving consistency? The author thinks the answer to this question is yes. However, any additional effort added to the survey will need to be evaluated closely. If too much effort is required to complete the survey, the respondent's patience may be over extended resulting in additional inconsistency. The author conjectures that on unfamiliar and innovative features a carefully designed preview of the treatments will provide better consistency.

If one can recognize inconsistency when the judgments are collected, can it be improved? Reevaluation or additional judgments can improve poor consistency.

However, will correcting the inconsistency bias the experiment or will one be ignoring the reason why some judgments are inconsistent? Additionally, it is not wise to make decisions based on single experiments where the factor significance is marginal. Even a powerful experiment conducted under different conditions can lead to different conclusions. Factors that possess marginal significance should always be suspect.

The graph in Figure 26 is a simple look at the factor levels when the response is sorted in descending order. All of the '+1' factor levels are shaded white and all of the '-1' factors levels are shaded black. One can immediately see the difference in the density of the shaded areas for factor 'A'. The '+1' levels of factor 'A' are associated with higher response levels. Factor 'A' is the background lighting color. The '+1' level of factor 'A' is the warmer color temperature. Any additional patterns are hard to discern. Possibly factor 'C' or perhaps the interaction effect AC (which is aliased with BD) is significant. As stated previously, factor 'B', 'accent lighting color', and its interactions should be viewed with caution because only one-half of the trials were performed with the accent lighting on.

Figure 26 Factors with the response sorted in descending order.

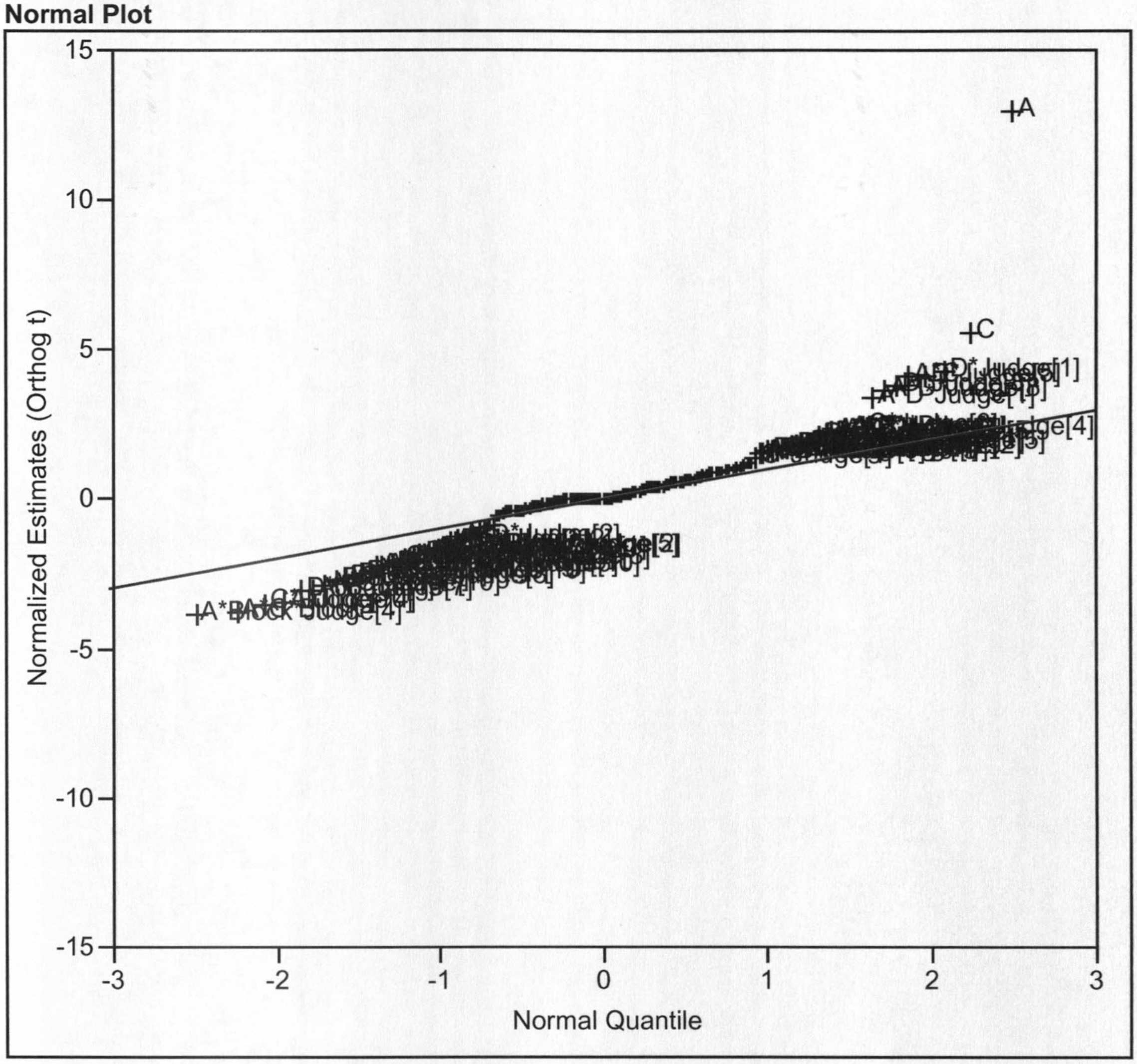


Source: Product work of the author.

Figure 27 is a normal probability plot of the factor effects. As suspected earlier, factor 'A' appears to be significant as it lies away from the pseudo random error line[3]. Factor 'C', background lighting intensity, brightness, also appears to be significant in moving the response. Respondents seem to prefer the brighter background lighting opposed to the dimmer background lighting. The partial pareto plot of the effect estimates in Figure 28 agrees with the normal probability plot. Factor effect 'A' is over two times more influential in moving the response than the nearest effect, factor 'C'. The effect estimate of factor 'C' is approximately 30% larger than next smallest effect, the interaction 'D*judge[1]', which as stated before, indicates a preference for the brighter background lighting. The remaining effect estimates are small and do not stand out from the adjacent effect estimates. However, these effects may still be real and not present due to chance.

[3] Lenth, Russel V. "Quick and Easy Analysis of Unreplicated Factorials." Technometrics 31 (1989): 469-473

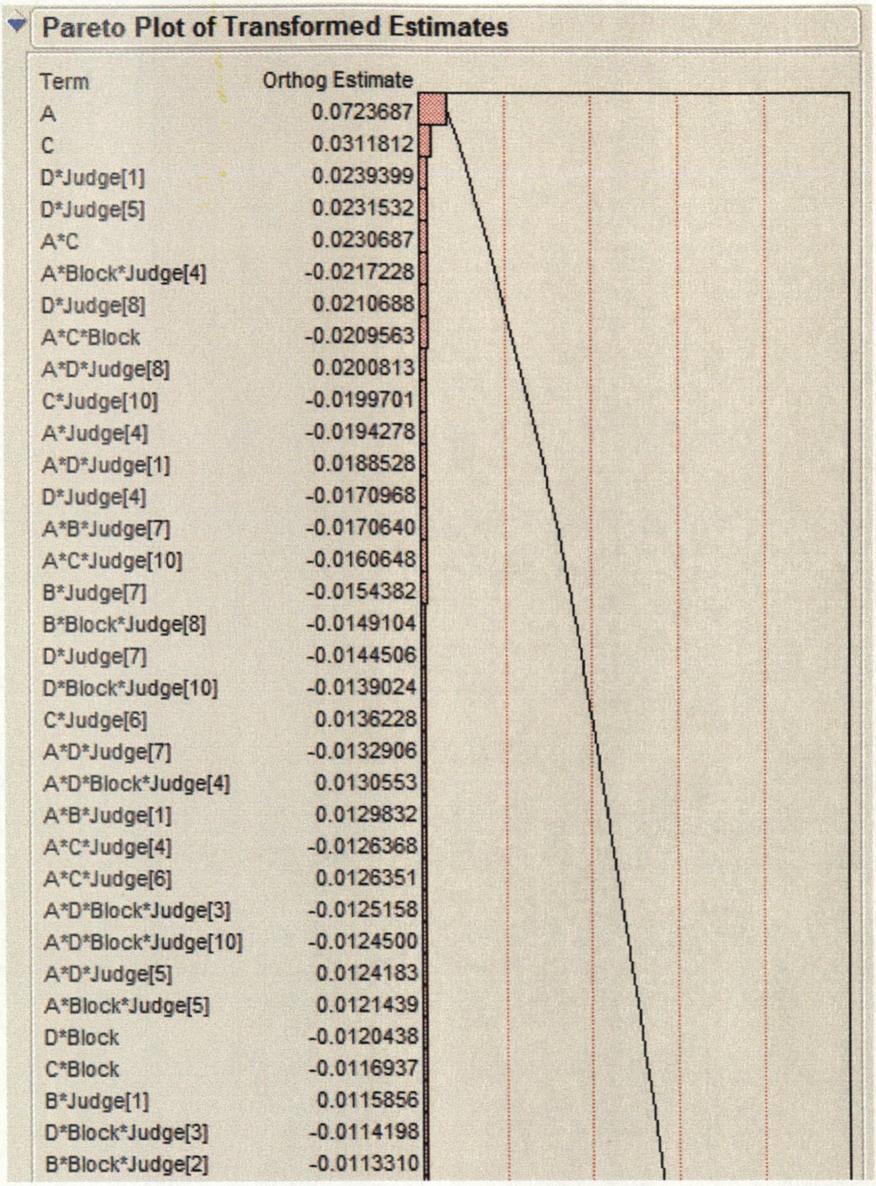
Figure 27 Normal plot of effects.



Blue line is Lenth's PSE, from the estimates population.

Source: Product work of the author.

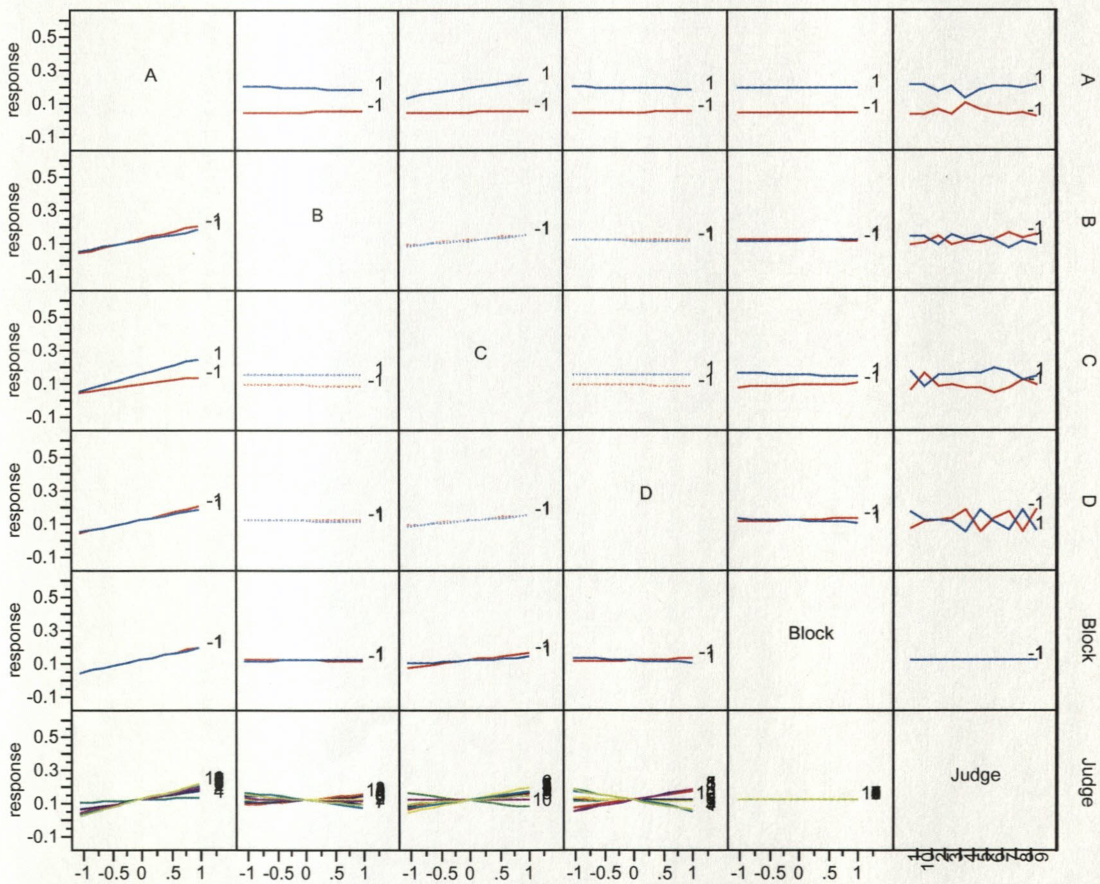
Figure 28 Pareto plot of effect estimates.



Source: Product work of the author.

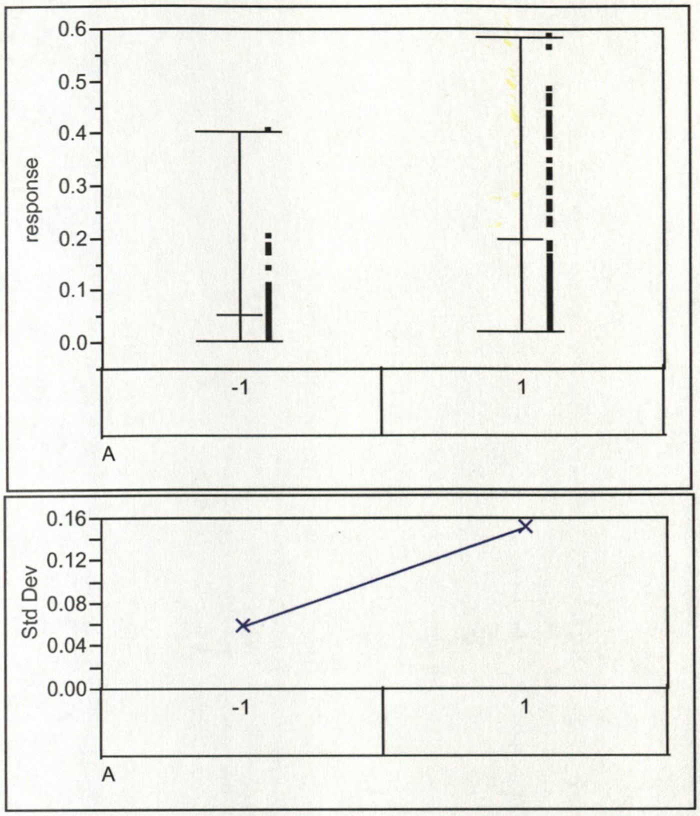
The interaction plots for two factor interactions are shown in Figure 29. The most interesting part of this figure is the right hand column where 'judge' is paired with the four main effects and the ambient blocking factor. As one would expect, the positive level of factor 'A', background lighting color, is always preferred regardless of the judge. A consensus among judges is almost achieved with factor 'C'. Two judges thought the opposite of the other eight judges when evaluating this factor. However, no consensus is reached with factors 'B' and 'D' (accent lighting color and intensity respectively). Everyone appears to have a different opinion about factors 'B' and 'D'. However, factor 'B' was examined separately and will be discussed in a later section of the paper due to only one-half of the trials being done with the accent lighting on. The variability chart in Figure 30 also supports the significance of factor 'A' as one can see the mean of the '+1' level of factor effect 'A' is higher than the mean of the '-1' level of factor effect 'A'.

Figure 29 Interaction profiles.



Source: Product work of the author.

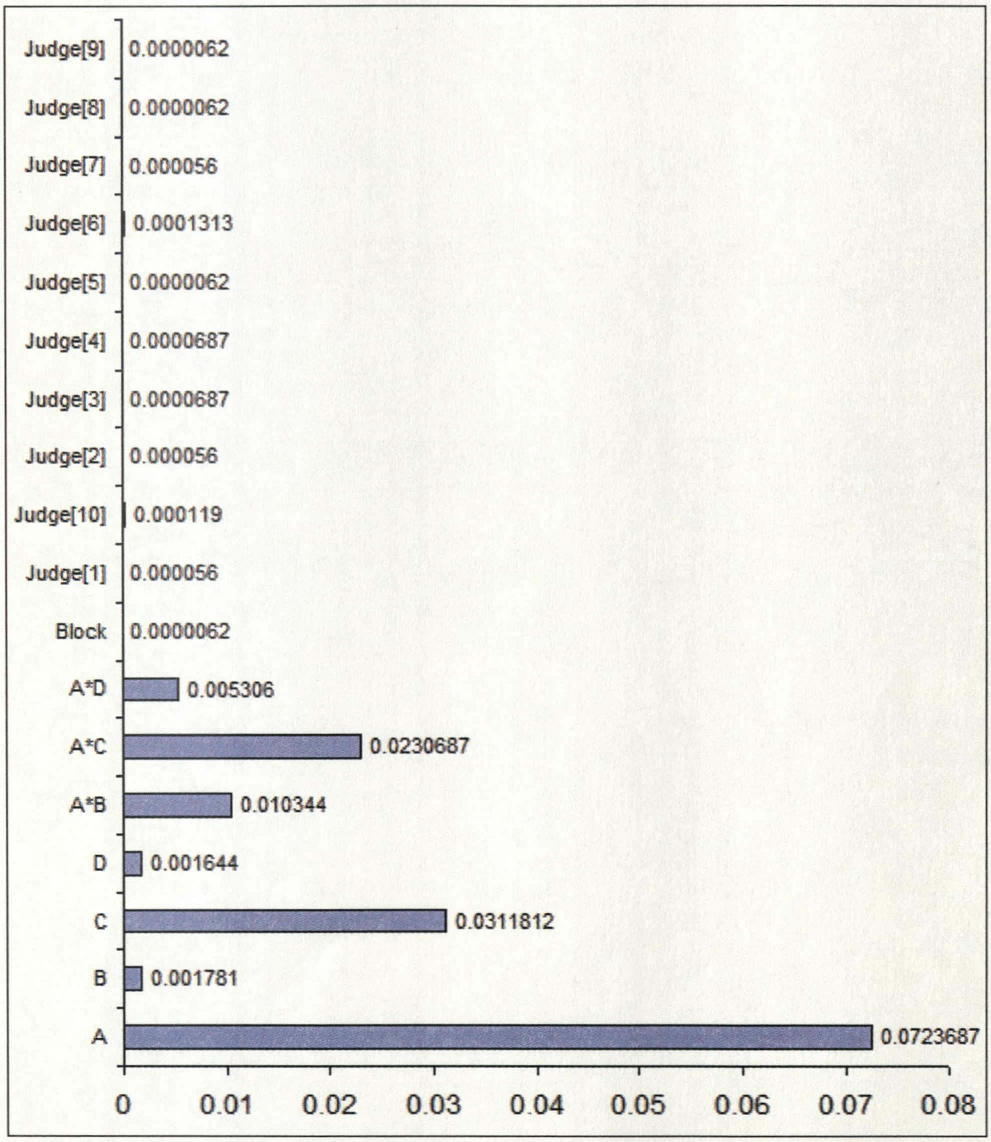
Figure 30 Variability chart for response grouped by A.



Source: Product work of the author.

Due to the normalization of ratings within runs, the effect estimates for the factors judge, block, and judge*block interactions are zero. For example, if one adds all of the responses for the '-1' blocks (ambient lighting), the sum is equal to 10. Conversely, if one add all of the responses for the '+1' blocks (ambient lighting), the sum is equal to 10. The normalization of ratings within runs causes the effect estimates for the blocking factors 'ambient lighting' and 'judge' to be zero. One can easily see this phenomenon in Figure 31. Figure 31 displays the absolute value of the effect estimates of the factors 'A', 'B', 'C', and 'D' along with their interaction effects. In the same chart above these effects are the absolute value of the effect estimates for the factors 'Block', 'Judge', and their interaction 'Block*Judge'. The effect estimates for the factors 'Block', 'Judge', and their interaction are all zero (except for rounding errors). However, a non-zero effect estimate is available as the factors 'judge' and 'Block' interact with the main effects 'A', 'B', 'C' and 'D'. For example, one may ask if the judge's preference for factor 'D' changes as the ambient lighting factor, 'Block' is changed from a well lit environment to a dark environment?

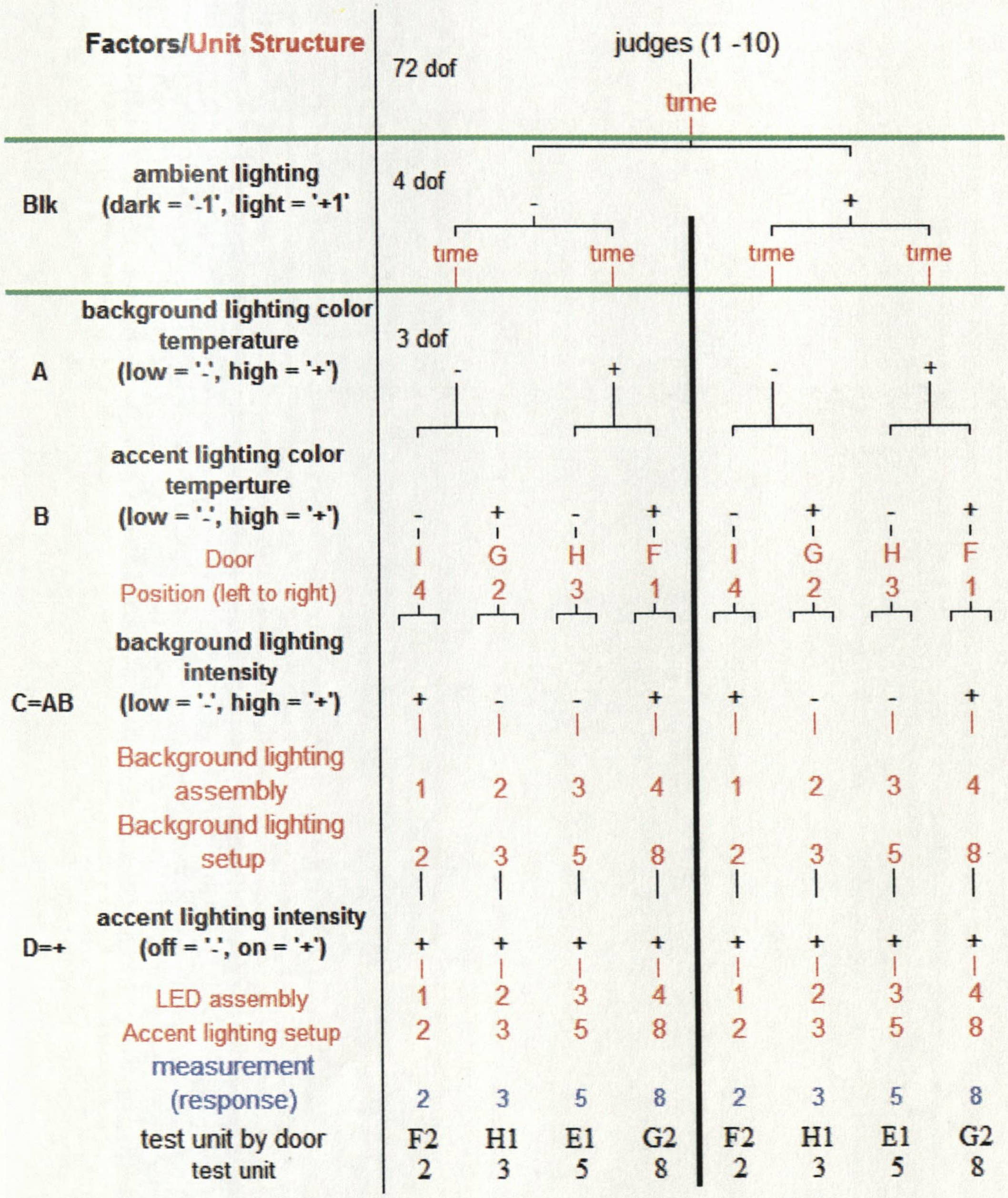
Figure 31 Scaled estimates.



Source: Product work of the author.

Figure 32 is the factor relationship diagram (FRD) of the experiment with only the factor 'D' +1 trials included. The experiment is a 2^{3-1} with a resolution of three. The main effect 'C' is aliased with the interaction 'AB'.

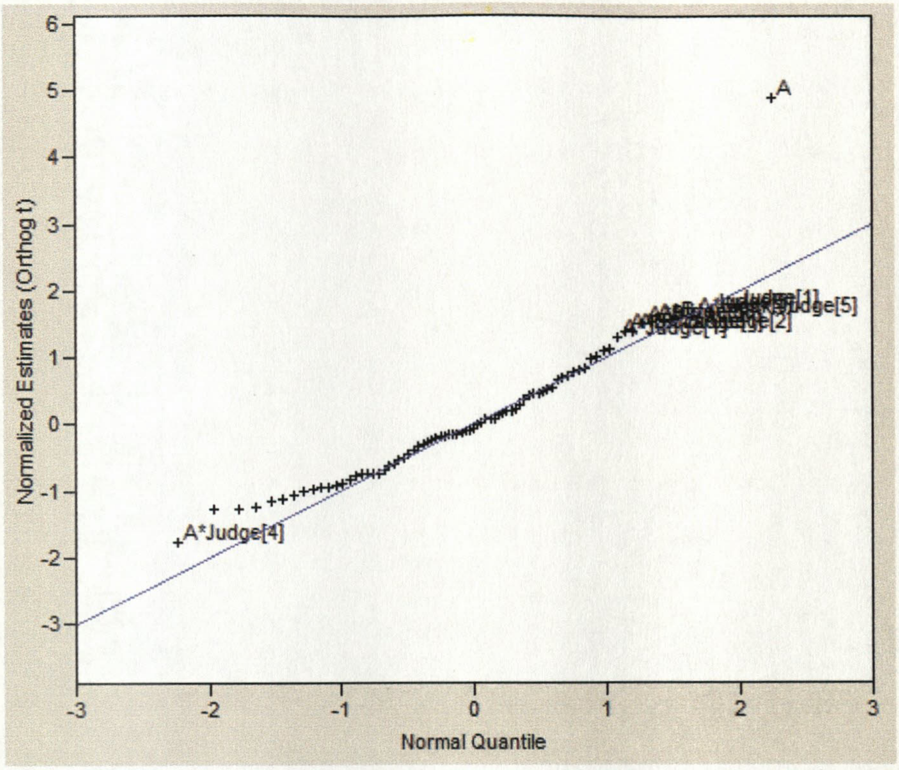
Figure 32 FRD with factor 'D' at '+1'.



Source: Product work of the author.

The results of the experiment differ very little when only the factor 'D' at +1 treatments are included. Factor 'A', background lighting color, is still significant. Figure 33 is the normal plot of this experiment.

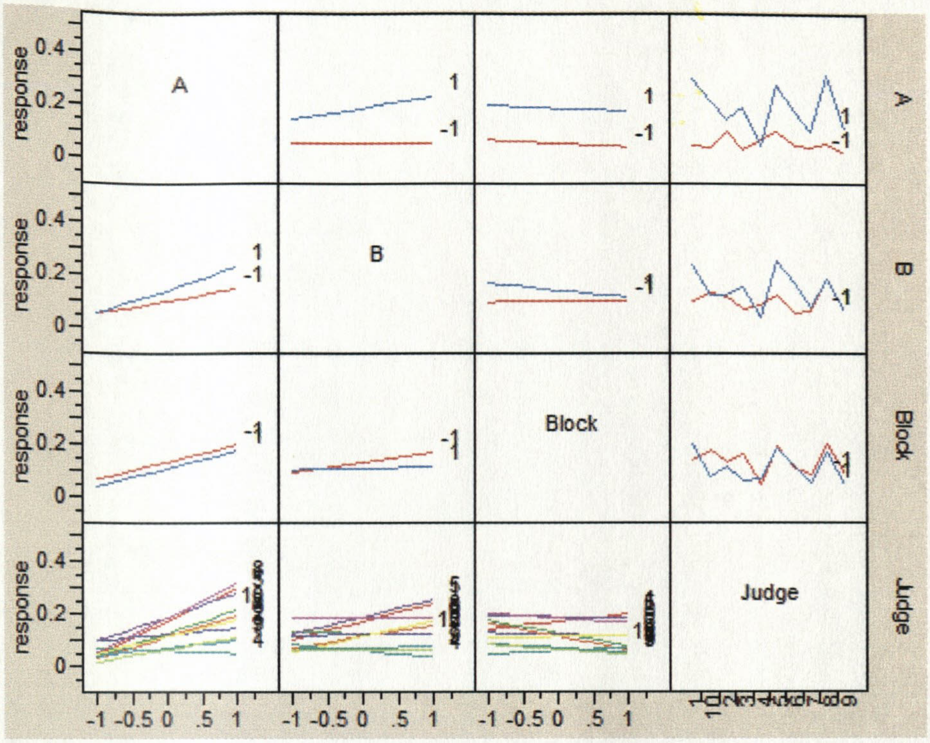
Figure 33 Normal plot of 'D' at +1.



Source: Product work of the author.

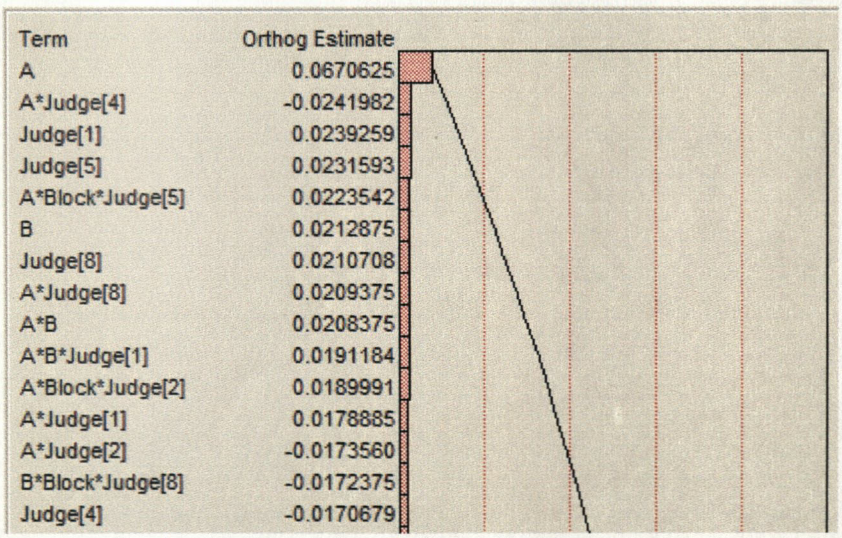
The interaction plots in Figure 34 do vary slightly regarding factor 'A'. The consensus regarding factor 'A' is less dramatic than in the previous analysis. However, if one compares the effect estimates for 'A', the difference is small. There is no consensus regarding factor 'B'. The interaction effect 'AB', which is aliased with factor 'C', can be examined in Figure 35, a pareto plot of the first 15 effect estimates. The author considered the interaction effect 'AB' not significant due to its close proximity with other effect estimates. The author also examined the factors 'judge' and 'block' in this experiment since their effect estimates are no longer driven to the value of zero. However, everything except the factor 'A' appears to have a weak or inconsistent influence on the response.

Figure 34 Interaction plots with 'D' at +1.



Source: Product work of the author.

Figure 35 Pareto plot of transformed estimates (first 15).



Source: Product work of the author.

One additional method to review the data is to analyze it by judge. This will allow one to gain some understanding into a individual judge's preference, however the author was interested in information regarding factors that the judges agreed upon. Therefore, this analysis was not conducted.

Conclusions and Recommendations

The experimental method used in this paper appears to be effective at gathering subjective information from the customer. Would one have reached a similar conclusion using an absolute scale, such as a Likert scale, instead of the relative comparison method used in the Analytical Hierarchy Process? The author believes that one may have obtained the same order of treatments. However, the differences between the treatments would not be useable. The author believes this is due to the difficulty of developing an internal standard when making judgments concerning a subjective and unfamiliar feature. Therefore, in the author's opinion the data obtained from an absolute scale would be unreliable. At the same time, the method used in this paper also appeared to have some difficulty due to the non-existence of a standard related to treatments used to conduct the refrigerator dispenser lighting experiment. However, the author still considered the data reliable and useable with most of the consistency ratios below the Saaty threshold of 0.10. The author concludes that an improvement in the consistency ratios is desired and conjectures that a preview of the treatments before the experiment may help the respondent establish an internal standard. However, the preview needs to be carefully designed so that the respondent's patience is not over extended.

Additionally, the author suggests that the surveys related to these types of experiments should be conducted by two people. The experiment conducted in this paper used only one person to administer the survey, collect response data, write down comments, and operate the fixture. Many comments were missed in this experiment. Copious notes are sometimes more valuable than response data. Additionally, the opportunity for an error, i.e. displaying the wrong treatment, was too great. If an error occurred while operating the fixture, it would likely increase the inconsistency. Unless the process is somehow recorded, this type of error is impossible to detect and correct.

The author recommends the development of more precise random consistency indices (RIs). The random consistency indices used in this paper appeared to be effective at gauging the consistency of the responses. However, a more precise and accurate random consistency indices would result in more confidence in the data.

Additionally, some research recommends that when random values are replaced with the discrete values in Saaty's scale, the center point of the interval should be generated using the geometric mean and not the arithmetic mean. The author used the arithmetic mean. The author thinks that the geometric mean may bias the results. For example, assume a random value of 4.48 is generated. When converted to a discrete value on Saaty's 1-to-9 scale, should this be a four or a five? If one uses the geometric mean as the center point, this value should be converted to a five. However, if one uses the arithmetic mean, this value should be converted to a four. All of these values are shown in Table 25. Due to the closeness of the geometric and arithmetic means, the author believes one could choose either method and the results will likely be the same.

Table 25 Geometric mean/arithmetic mean.

Saaty's Scale	Geometric mean	Arithmetic mean
1		
	1.41	1.50
2		
	2.45	2.50
3		
	3.46	3.50
4		
	4.47	4.50
5		
	5.48	5.50
6		
	6.48	6.50
7		
	7.48	7.50
8		
	8.49	8.50
9		

Source: Product work of the author.

One should understand which factors would have an estimated effect of zero due to the normalization process. One should look for these factors in the interactions with the main effect whose effect estimates are not normalized to zero. If this is unacceptable, one should redesign the experiment.

All of the judges in this experiment preferred the background lighting with the warmer color temperature, regardless of the ambient lighting conditions. Most judges appeared to prefer to the brighter background lighting. However, this factor's significance is not overwhelming. Therefore, additional experimentation is required to confirm these findings. As represented in this experiment, the judges did not reach a consensus regarding the accenting lighting intensity or color. Additional insight may be forthcoming if the data were analyzed by 'judge'.

References

The American Supplier Institute (ASI) QFD page. 2000. The American Supplier Institute. 21 December 2006 <http://www.amsup.com/qfd/index.htm>.

Barzilai, Jonathan. "Notes on the Analytic Hierarchy Process." Proceeding of the NSF Design and Manufacturing Research Conference (2001): 1-6.

Bergerud, Wendy A. "Displaying Factor Relationships in Experiments." Biometrics Information Pamphlet No. #55 15 November 1996.

Birren, Faber. Color Psychology and Color Therapy, a Factual Study of the Influence of Color on Human Life. Secaucus, N.J.: The Citadel Press, 1950.

Chartrand, Gary. Introductory Graph Theory. New York: Dover Publications, 1977.

Franceschini, Fiorenzo. Advanced Quality Function Deployment. Boca Raton: CRC Press LLC, 2002.

Forman, Ernest H., "Random indices for incomplete pairwise comparison matrices." European Journal of Operational Research 48 (1990): 153-155.

Forman, Ernest H., with Gass, Saul I, "The Analytic Hierarchy Process – An Exposition.", Operations Research vol.49, no. 4, (July-Aug 2001): 469-486.

Houston, Archester. “Survey Handbook.” Organizational Systems Division, Total Quality Leadership Office, Office of the Under Secretary of the Navy. January 2007 <http://unpan1.un.org/intradoc/groups/public/documents/aspa/unpan002507.pdf>.

Ishizaka, Alessio, Dieter Balkenborg, and Todd Kaplan. “AHP Does Not like Compromises: the Role of Measurement Scales.” Audencia Nantes School of Management web site siteAudi January 2007 <http://www.audencia.com/fileadmin/images_site/PDF_WORD/Alessio_Ishizaka/Graz.pdf >.

iSixSigma definition page for Voice of the Customer(VOC). Posted By: Terry, Modified By: Alice Gobeille, Last Modified: Aug. 20, 2003. 21 December 2006 <http://www.isixsigma.com/dictionary/Voice_Of_the_Customer_VOC-391.htm>.

Kruithof, A. A. Tubular Luminescence Lamps for General Illumination," Philips Technical Review 6 (1941): 65-96.

Lenth, Russell V. “Quick and Easy Analysis of Unreplicated Factorials.” Technometrics 31 (1989): 469 – 473.

"Introduction to Retail Lighting." Lighting Design Lab web site. January 2007. <http://www.lightingdesignlab.com/articles/retail/intro_retail.htm>.

"The Measurement Process." 30 November 2006. Professor Punj's web page on the University of Connecticut libraries web site. January 2007. <http://www.lib.uconn.edu/~punj/m35010.pdf>

Montgomery, Douglas C. Design and Analysis of Experiments. 5th ed. New York: John Wiley & Sons, 2001.

Nishizawa, Kazutomo. "Estimation of Unknown Comparisons in Incomplete AHP and It's Compensation." Report of the Research Institute of Industrial Technology. Nihon University 77 (2005): 1.

Poyhonen, Mari A., Raimo P. Hamalainen, and Ahti A. Salo. "An Experiment on the Numerical Modeling of Verbal Ratio Statements." Journal of Multi-Criteria Decision Analysis. 6 (1997): 1-10.

Saaty, Thomas L. Fundamentals of Decision Making and Priority Theory. Pittsburg: RWS Publications, 2000.

Saaty, Thomas L. Decision Making For Leaders. Pittsburg: RWS Publications, 2001.

Setiawan, Sugeng., "The Effect of Initial Selections in Estimating the Missing Comparisons in an imcomplete AHP Matrix." Thesis. Louisiana State University, 2002.

"Small Retail Lighting Know how." The DesignLights™ Consortium (DLC) web site. January 2007. <http://www.designlights.org/downloads/retail_guide.pdf>.

Stevens, S.S. (1946). "On the theory of scales of measurement." *Science*. 103 (1946): 677-680.

Velleman, Paul F., and Leland Wilkinson. "Nominal, Ordinal, Interval, and Ratio Typologies are Misleading." The AmericanStatistician.47:1 (1993): 65-72.

Weintraub, Steven. "The Color of White: Is there a "preferred" color temperature for the exhibition of works of art?" Western Association for Art Conservaton 'WAAC' Newsletter. 21.3 (September 1999). 28 December 2006 <http://palimpsest.stanford.edu/waac/wn/wn21/wn21-3/wn21-308.html>.

Wikipedia, The Free Encylclopedia, measurement page. 20:58, 14 December 2006. Wikimedia Foundation, Inc. 21 December 2006 <http://en.wikipedia.org/wiki/Measurement>.

Zimmerman, E., Linda Steinmann, and Vince Schueler, "Designing Customer Survey That Work." Quality Digest web site. January 2007. <http://www.qualitydigest.com/oct96/surveys.html>.

Appendix

Appendix 1

Generating Random n-1 Comparison Schedules

The minimum number of comparisons required for a comparison schedule is n-1, where n is the number of alternatives. For example, a minimal comparison schedule consisting of eight alternatives requires seven independent comparisons[4]. In a minimum comparison schedule, there exists only one method to calculate the numerical relationship between any two alternatives.

In a minimal comparison schedule consisting of eight alternatives, one has 8! or 40,320 possible minimal comparison schedules to choose from. There are many ways to choose a minimal comparison schedule. Setiawan investigates five methods in his research on selecting initial comparisons (Setiawan, 2002):

1. Basing all comparison on one alternative
2. Arranging alternatives in decreasing order of their weights and then selecting comparisons from adjacent alternatives.
3. Generating comparisons randomly (ensuring that the comparison schedule is connected)
4. Comparisons with the highest a_{ij} values are selected.
5. Comparisons are ranked starting with the highest value and then the median is selected as the first comparison.

Setiawan's research investigates which selection method produced the most accurate estimation of the ranking of alternatives. Setiawan's research shows that for matrices or order 10, 15, and 20, there is practically little difference between the best method, basing all comparisons on one alternative, and selecting the comparisons

[4] Independent comparisons are non-redundant

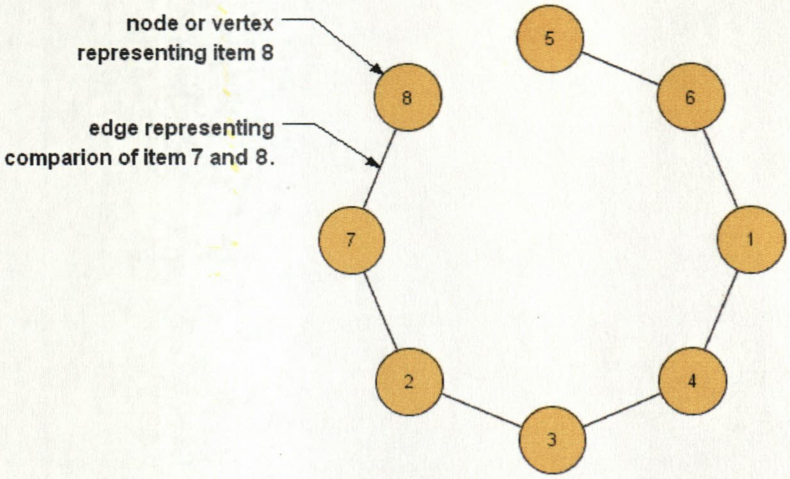
randomly. Moreover, the author sees little practical difference in all of the methods trialed by Setiawan. However, the author did not base the decision to generate the comparison schedules randomly only on the requirement of 'accurate ranking of alternatives' but on two equally if not more important criteria.

The respondent possessed no initial knowledge of the alternatives and viewed the alternatives for the first time during the survey. For the respondent to gain any knowledge concerning the alternatives would have required more time to administer the survey, which the author feels was not practical given the length of the survey. Require more time from a respondent may increase the likelihood of errors and requires additional resources.

Eliminating any potential bias due to the question set or question set order was the most important criteria. If the experiment was conducted in such a way that randomized question sets were not used, bias may have been introduced. Furthermore, this type of bias is impossible to detect after the experiment, therefore one cannot compensate for it. The experiment was conducted such that each participant was given a different randomly generated comparison schedule. Therefore, any bias associated with the question set or question set order was 'averaged out'. This appendix discusses an algorithm to generate random comparison schedules.

Comparison schedules are conveniently represented using a list of comparisons or a graph. For example, the list of comparisons (5,6), (6,1), (1,4), (4,3), (3,2), (2,7), and (7,8) are also represented in the graph shown in Figure 36. Each ***node*** or ***vertex*** represents an item that is compared and each ***edge*** represents a comparison between the two vertices that it connects.

Figure 36 Graph representation of comparison schedule.



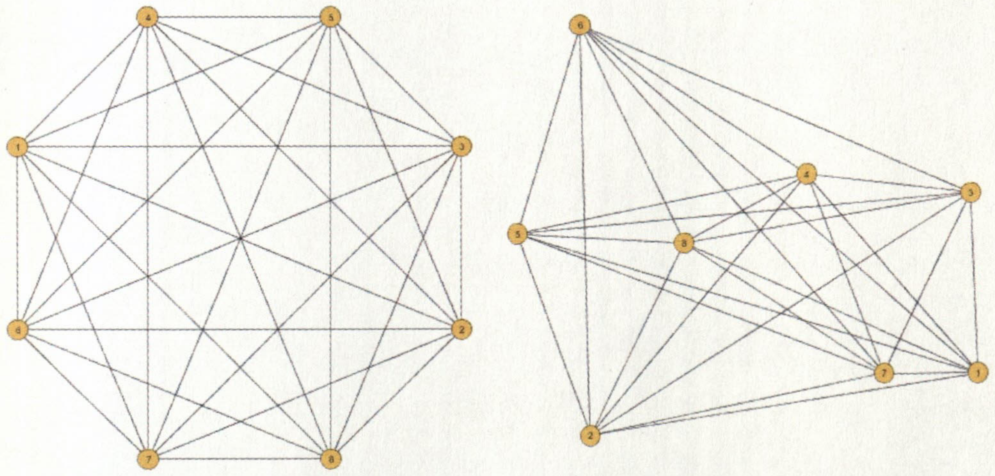
Source: Product work of the author.

The modeling of comparison schedules with a graph allows one to borrow several concepts from graph theory. These concepts are used to produce an algorithm that randomly generates the desired comparison schedules. Before discussing the algorithm that generates the comparison schedules, one must first understand some elementary concepts concerning graph theory.

Put very simply, a graph is a set of vertices connected by a set of edges. Circles or dots represent the vertices, and the connections between vertices are represented by edges that are drawn as straight lines or arcs. The edges in a graph can be undirected or directed depending on the relationship that exists between the vertices. If there exists a one-way relationship between two vertices then a directed edge is required to properly represent that relationship. A graph that utilizes such relationships is a directed graph. We are only concerned with undirected graphs. Therefore, the edges are represented by lines with no arrows or directional features.

The spatial orientation of the vertices and edges in a graph typically contains no information and is usually drawn in a fashion that best illustrates the relationships to the observer. Generally, all of the information contained in a graph is contained in the relationships of the vertices, and these relationships are represented by the edges connecting them. For example, Figure 37 contains two different representations of the same graph. The two graphs appear to be very different, but the relationships of the vertices are the same. The first representation is more orderly and one can immediately discern several properties about the graph that the second representation obviously possesses but does not immediately disclose.

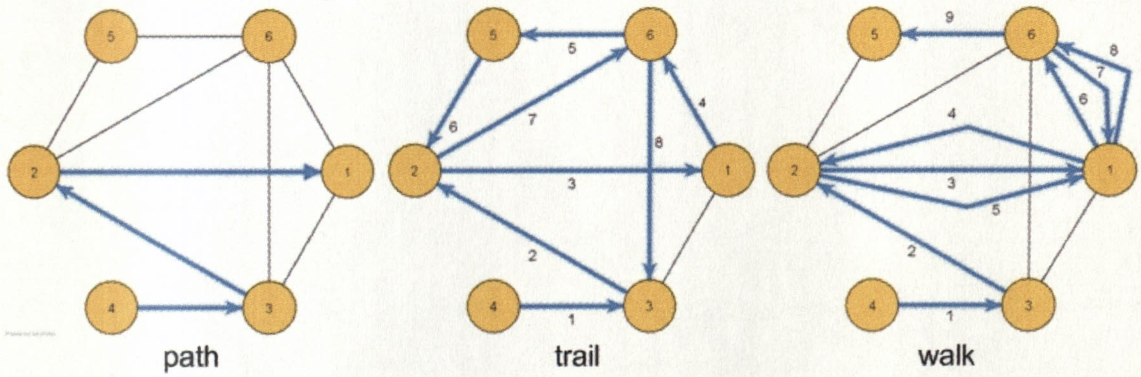
Figure 37 Two representations of the same graph.



Source: Product work of the author.

Like any branch of mathematics, graph theory possesses its own set of terminology to make discussions more concise and efficient. A ***walk*** is an alternating of vertices and edges that begins at a vertex and ends at a vertex. An edge or a vertex need not be unique in a walk, i.e. an edge or a vertex can be visited more than once in a walk. A ***trail*** is a walk in which no edge is repeated, but vertices can be repeated. A ***path*** is a walk in which no edges or vertices are repeated. See Figure 38 for examples of a path, trail, and a walk. The numbers along the edges of the trail and walk are included to indicate the sequence followed. Arrows have been added to the lines to better illustrate the direction of the sequence and do not indicate any kind of directional relationship between the vertices. The path from vertex 4 to vertex 1 has a length of 3, the trail from vertex 4 to vertex 3 has a length of 8, and the walk from vertex 4 to vertex 5 has a length of 9. Two paths are equal if they traverse the same vertices and edges in the same sequence. Equivalence for trails and walks is defined similarly.

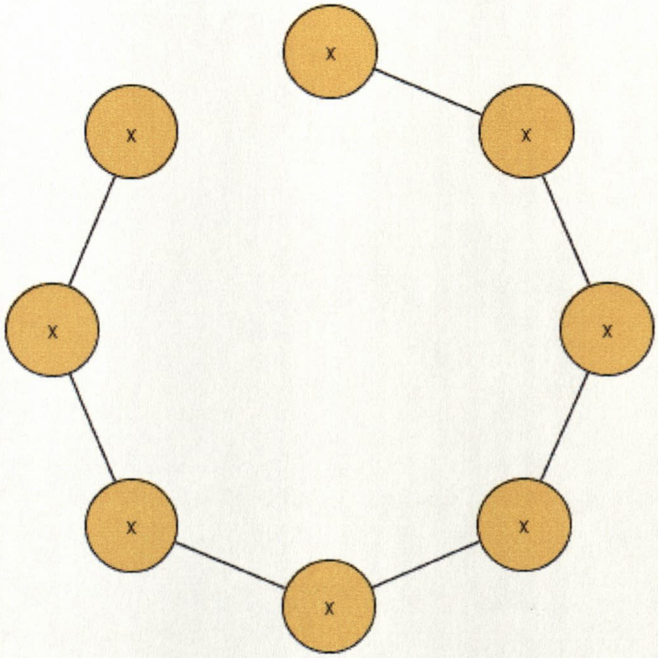
Figure 38 Path, trail, and walk examples.



Source: Product work of the author.

A trail in which the beginning and ending vertex are the same and contains at least three edges is called a ***circuit***. A circuit, which does not repeat any vertices, except the first and last, is called a ***cycle***. The ***degree of a vertex*** is the number of edges incident on that vertex. The ***degree of a graph*** is the number of vertices that it contains and its ***size*** is the number of edges it contains. Two vertices in a graph are ***connected vertices*** if a path exists between them. A graph is ***connected graph*** if every pair of vertices is connected. In Figure 38, the example of the path connects vertices 1 and 4 by the path 1, 2, 3, and 4. A graph with no cycles is called an ***acyclic graph*** or a ***forest***. A ***tree*** is defined as a connected acyclic graph. The author is interested in comparison schedules that can be represented by acyclic connected graphs or trees of degree eight and size of seven. Figure 39 is a generic representation of this type of tree. The 'x's represent the number of the item to be compared. As one can see, there is only one path to get from one vertex to another, therefore there is only one-way to obtain a numerical relationship between two objects using transitivity. This condition also guarantees that the comparisons are non-redundant (independent).

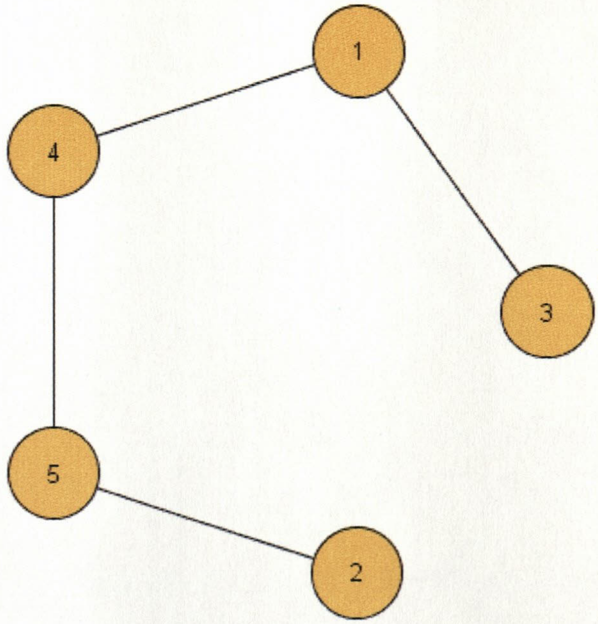
Figure 39 Eight item tree with seven edges.



Source: Product work of the author.

An adjacency matrix is a non-graphical method of representing a graph. An adjacency matrix is an 'nxn' matrix with the rows and columns representing the vertices. The number one is placed in every cell that corresponds to vertices that are connected by a path length of one. Zeroes are placed in all the remaining cells. For example, the graph in Figure 40 has five vertices with four paths of length one between them. As shown in the adjacency matrix of Figure 41, there are five number ones placed in the appropriate cells. One can also view the adjacency matrix as all walks that are of length one. For example, there exists one walk of length one between vertex pairs (3,1), (1, 4), (4,5), and (5,2). These are highlighted in yellow in the M^1 matrix of Figure 42.

Figure 40 Example graph.



Source: Product work of the author.

Figure 41 Adjacency matrix.

	1	2	3	4	5
1	0	0	1	1	0
2	0	0	0	0	1
3	1	0	0	0	0
4	1	0	0	0	1
5	0	1	0	1	0

Source: Product work of the author.

Figure 42 Adjacency matrix and its powers.

M^1

	1	2	3	4	5
1	0	0	1	1	0
2	0	0	0	0	1
3	1	0	0	0	0
4	1	0	0	0	1
5	0	1	0	1	0

M^2

	1	2	3	4	5
1	2	0	0	0	1
2	0	1	0	1	0
3	0	0	1	1	0
4	0	1	1	2	0
5	1	0	0	0	2

M^3

	1	2	3	4	5
1	0	1	2	3	0
2	1	0	0	0	2
3	2	0	0	0	1
4	3	0	0	0	3
5	0	2	1	3	0

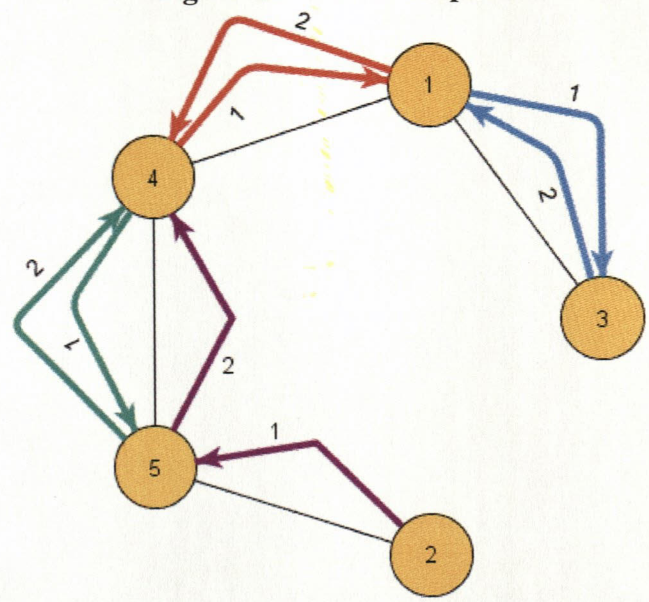
M^4

	1	2	3	4	5
1	5	0	0	0	4
2	0	2	1	3	0
3	0	1	2	3	0
4	0	3	3	6	0
5	4	0	0	0	5

Source: Product work of the author.

An amazing property of the adjacency matrix is that the walk length relationship is still valid when the adjacency matrix is raised to some power. When the adjacency matrix is squared the result is a matrix that indicates all walks in the graph that are of length two. Since a walk of length two exists between every vertex and itself, the square of the adjacency matrix also yields the degree of the all vertices along its diagonal. One must remember that a walk is an alternating sequence of the vertices and edges and the edges and vertices need not be unique. Figure 43 displays several walks that are contained in the second power of the adjacency matrix in Figure 41 as matrix M^2. The red arcs represent one walk of length two from vertex four to vertex four and the green lines represent the other walk of length two from vertex four to vertex four. This is in agreement with the M^2 matrix in Figure 42 where the intersection of row four and column four contains the number two. It can be seen that the same edge is traversed twice in each walk. Since vertex three is of degree one, it only has one walk of length two. As mentioned before, the number of walks of length two from a vertex back to that same vertex is the degree of that vertex. Also shown in Figure 43 in purple is the walk of length two from vertex two to vertex four. The M^3 matrix in Figure 42 displays the number of walks of length three and the M^4 matrix in Figure 42 displays the number of walks of length four.

Figure 43 Walk examples.



Source: Product work of the author.

For the purposes of the scheduling generating algorithm, the author is interested in paths, not walks. The number of paths in a tree is found by calculating all of the powers of the adjacency matrix up to the number of edges (n-1 vertices) and then recording all of the walks of length one that are in the upper half or bottom half of the matrices excluding the diagonal. For example, the graph shown in Figure 40 has the adjacency matrix and its powers up to (n-1) vertices shown in Figure 42. If one records the number of single paths in the upper half of all the matrices there are ten paths or $\binom{5}{2}$.

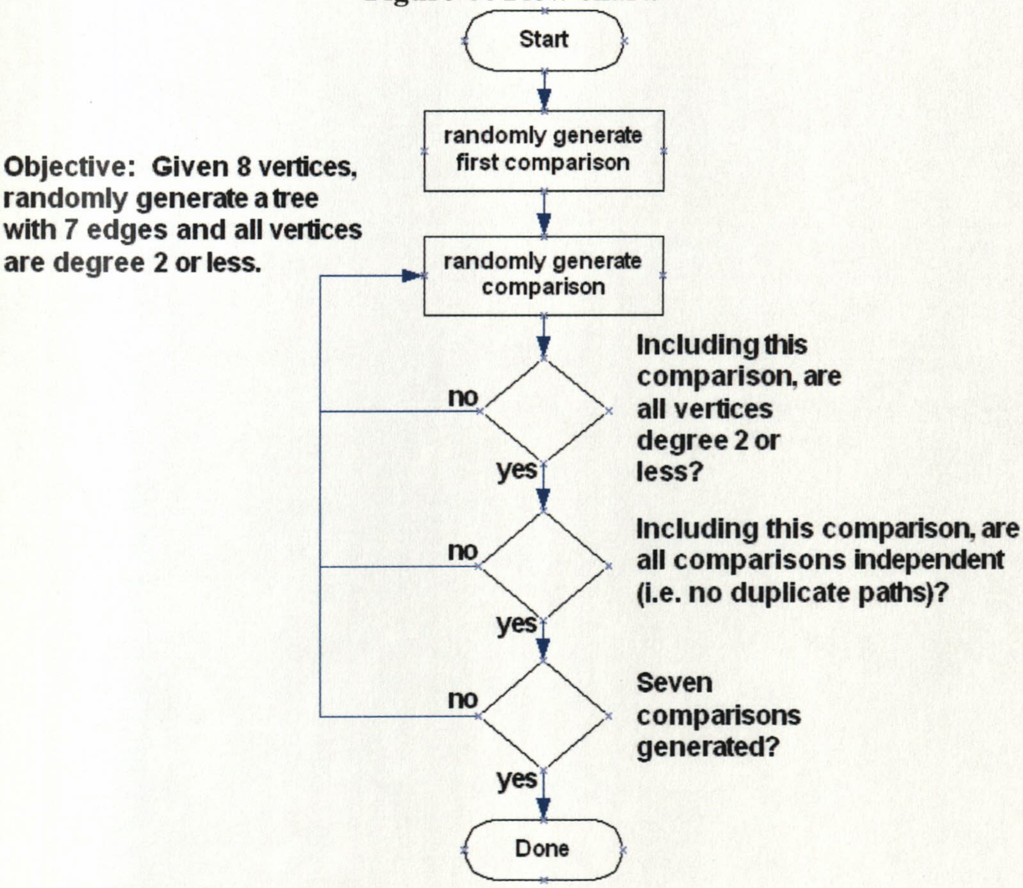
These paths represent all the possible vertex combinations.

The basic flowchart representing the algorithm is shown in Figure 44. The first step is to generate the first comparison randomly by arbitrarily selecting two vertices. We will be completing the top half of the adjacency matrix so the second value (the column value) in the comparison should always be greater than the first (the row value). We only complete the top half because the bottom half is a mirror image of the top half (symmetry) and the diagonal is filled with zeros since there are no loops. After the first comparison is generated, it is recorded.

Next, the second comparison is randomly generated just as the first. One then calculates the degree of all vertices and records all paths. Then one must ask two questions. Is the degree of all vertices two or less (this will always be true with only two comparisons)? Are all paths unique (Only one path exists at this point, so the only duplicate is the path generated by the first comparison)? If these two conditions are satisfied, the comparison is valid and can be recorded. One can then generate the third comparison just as the first comparison was generated. Again, calculate the degree of all vertices and record all paths. Is the degree of all the vertices two or less and are all paths unique? If these two conditions are satisfied, one can record the third comparison and move on to generate the fourth comparison and so on. This process is repeated until there are seven comparisons that satisfy the two requirements: 1. All vertices are of degree two or less 2. All comparisons are independent.

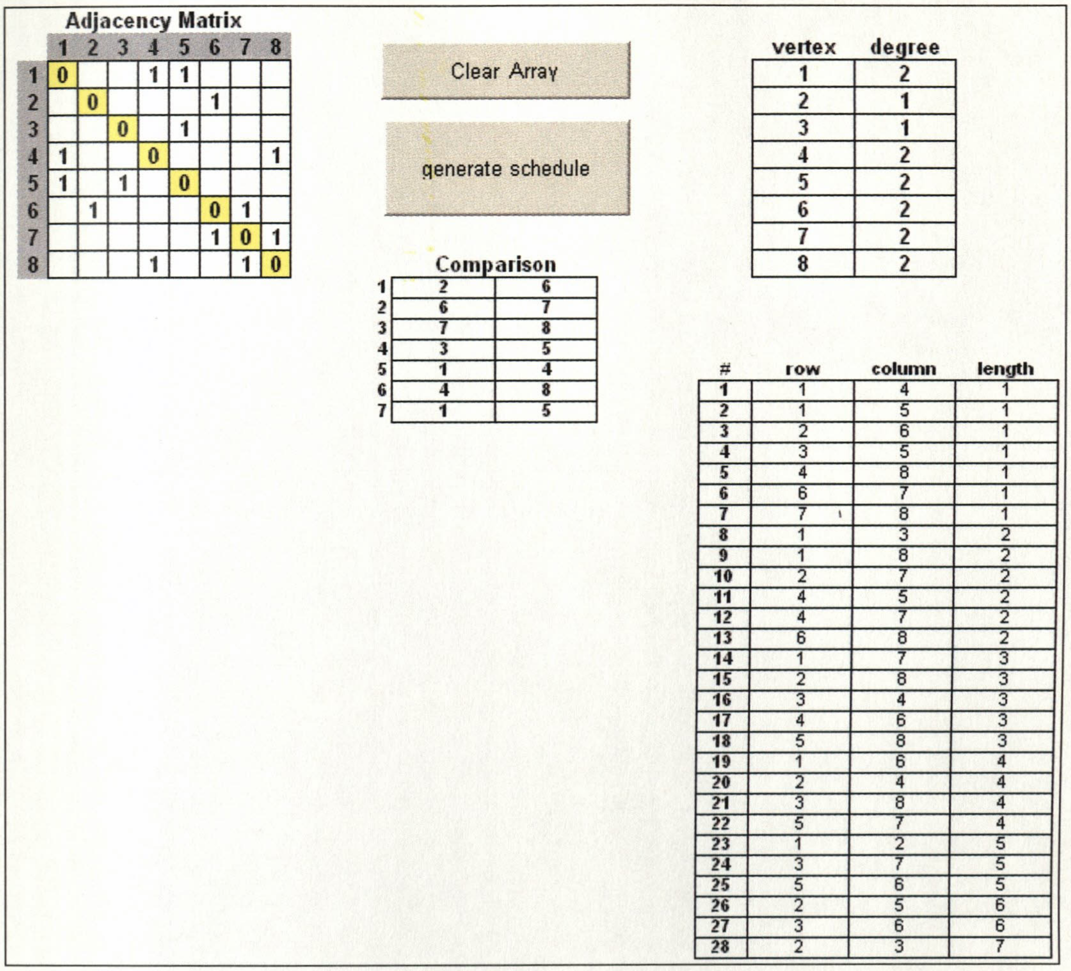
The author implemented this algorithm using visual basic for applications in Microsoft Excel 97. See Figure 45 for a screen shot of the spreadsheet application. To generate the schedule, one must first click the '*clear array*' button. This clears all the values in the spreadsheet. After the array is cleared, one clicks the '*generate schedule*' button to populate the spreadsheet as shown. The program generates four tables. The adjacency matrix is located in the upper left corner. The vertex list with their degrees is located in the upper right corner. In the middle of the screen is the comparison schedule shown as pairs of vertices. The first column represents the row and the second column represents the column. The path list is shown in the table located at the bottom right of the screen. There are 28 paths for eight vertices (one path of length 7, 2 paths of length 6, 3 paths of length 5, 4 paths of length 4, 5 paths of length 3, 6 paths of length 2, and 7 paths of length 1). The key to implementing the algorithm is the generation of the adjacency matrix and its powers up to the number of vertices minus one. This process generates the necessary information to check the degree of the vertices and redundancy of paths (independence of comparisons). The code is contained in Appendix 2.

Figure 44 Flow chart.



Source: Product work of the author.

Figure 45 Screen shot of schedule generating software.



Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0			1	1			
2		0				1		
3			0		1			
4	1			0				1
5	1		1		0			
6		1				0	1	
7						1	0	1
8				1			1	0

vertex	degree
1	2
2	1
3	1
4	2
5	2
6	2
7	2
8	2

Comparison

1	2	6
2	6	7
3	7	8
4	3	5
5	1	4
6	4	8
7	1	5

#	row	column	length
1	1	4	1
2	1	5	1
3	2	6	1
4	3	5	1
5	4	8	1
6	6	7	1
7	7	8	1
8	1	3	2
9	1	8	2
10	2	7	2
11	4	5	2
12	4	7	2
13	6	8	2
14	1	7	3
15	2	8	3
16	3	4	3
17	4	6	3
18	5	8	3
19	1	6	4
20	2	4	4
21	3	8	4
22	5	7	4
23	1	2	5
24	3	7	5
25	5	6	5
26	2	5	6
27	3	6	6
28	2	3	7

Source: Product work of the author.

Appendix 2

Excel Visual Basic Code to Produce Random (n-1) Comparisons

```
Option Base 1
Private Sub Clear_Array_Click()
' Sets entire array to null
Range("Schedule_Array").Value = Null
' Fills in the diagonal with 0s
For i = 1 To 8 Step 1
   Range("Schedule_Array").Cells(i, i).Value = 0
Next
Range("Schedule_Array").Cells(1, 1).Value = 0
Worksheets("Sch_Gen").Range("P4:Q12").Value = Null
Worksheets("Sch_Gen").Range("L12:M19").Value = Null
Worksheets("Sch_Gen").Range("p17:r44").Value = Null
End Sub
Private Sub gen_schedule_Click()
Dim Random_Value_1, Random_Value_2 As Double
Dim Schedule_Array(1 To 7, 1 To 2) As Integer ' 1 length paths, will be 7
Erase Schedule_Array
Dim Schedule_Array_Size As Integer
Dim path_list(1 To 28, 1 To 3) As Integer
Erase path_list
Dim path_list_row As Integer
Dim path_list_length As Integer
Dim vertex_degrees(1 To 8, 1 To 2) As Integer
Erase vertex_degrees
Dim number_vertices As Integer
Dim in_list As Boolean

' M1(1, i, j) is the adjacency matrix of the graph
' M1(2, i, j) is the adjacency matrix squared
' M1(3, i, j) is the adjacency matrix cubed, etc.
Dim M1_Array(1 To 7, 1 To 8, 1 To 8) As Integer

' First value in Two_Arc_Array array contains the size
' Starting with 2nd row, col 1 contains pivot,
' col 2 contains row, col 3 contains col
Dim value_not_duplicate As Boolean
Dim value_independent As Boolean
Dim vertices_under_2 As Boolean
Dim value_acceptable As Boolean
Dim k As Integer ' used in the Two_Arc_Array routine

number_vertices = 8 ' 8 items to compare

' ------------------------------------------
' Generate first value for first cell in array
Randomize    ' Initialize random-number generator.
Random_Value_1 = Int((7 * Rnd) + 1)   ' Generate random value between 1 and 8.

' generate second value for first cell in array
' Puts second value generated so that coordinate is locatated in the upper half
' of a diagonal matrix i.e. value must not be equal or less than Random_Value_1
Random_Value_2 = 0
While (Random_Value_2 <= Random_Value_1)
Randomize
```

```
Random_Value_2 = Int((8 * Rnd) + 1)
Wend

' populate adjacency matrix
M1_Array(1, Random_Value_1, Random_Value_2) = 1
M1_Array(1, Random_Value_2, Random_Value_1) = 1
Range("Schedule_Array").Cells(Random_Value_1, Random_Value_2).Value = 1
Range("Schedule_Array").Cells(Random_Value_2, Random_Value_1).Value = 1
Schedule_Array(1, 1) = Random_Value_1
Schedule_Array(1, 2) = Random_Value_2
Worksheets("Sch_Gen").Cells(12, 12).Value = Random_Value_1
Worksheets("Sch_Gen").Cells(12, 13).Value = Random_Value_2

Schedule_Array_Size = 1

' ---------------------------------------------
' Generate second comparison in the comparison schedule
' Can be any row and column as long as
' 1. it is not a duplicate
' 2. any vertex does not exceed a degree of two
' 3. vertexes in new comparison are not already connected i.e. comparison
'   be independent
' Set value_acceptable to false to start while loop

For z = 1 To 6 Step 1 ' generate the next six comparisons

value_acceptable = False

While (value_acceptable = False)

  value_not_duplicate = True ' set to false if duplicate found
  value_independent = True ' set to false if match found in Two_Arc_Array array
  vertices_under_2 = True ' set that all vertices are 2 degrees are less

  Randomize   ' Initialize random-number generator.
  Random_Value_1 = Int((7 * Rnd) + 1)   ' Generate random value between 1 and 8

  'generate second value for second cell in array
  'value must not be equal to Random_Value_1 or less than Random_Value_1
  ' i.e. it must be greater than Random_Value_1
  Randomize   ' Initialize random-number generator.
  Random_Value_2 = 0
  While (Random_Value_2 <= Random_Value_1)
  Randomize
  Random_Value_2 = Int((8 * Rnd) + 1)
  Wend

  ' Is new compariso acceptable?

  ' Is new comparison a duplicate?
  For i = 1 To Schedule_Array_Size Step 1
    If ((Schedule_Array(i, 1) = Random_Value_1) And _
    (Schedule_Array(i, 2) = Random_Value_2)) Then
    value_not_duplicate = False
    End If
  Next
```

```
    ' Is comparison independent?
    ' Is the new comparison already in the path list
    For i = 1 To path_list_length
        If ((path_list(i, 1) = Random_Value_1) And _
            (path_list(i, 2) = Random_Value_2)) Then
                value_independent = False
        End If
    Next

    ' Does new value cause any of the previous vertices to
    ' exceed a degree of three?
    ' If so, new comparison will not be acceptable
    For i = 1 To number_vertices Step 1
        If (((Random_Value_1 = vertex_degrees(i, 1)) Or _
            (Random_Value_2 = vertex_degrees(i, 1))) And _
            (vertex_degrees(i, 2) = 2)) Then
            vertices_under_2 = False
        End If
    Next

    ' Is value acceptable
    If ((value_not_duplicate = True) And _
        (value_independent = True) And _
        (vertices_under_2 = True)) Then
    value_acceptable = True
    End If

Wend


Schedule_Array_Size = Schedule_Array_Size + 1


' populate adjacency matrix
Range("Schedule_Array").Cells(Random_Value_1, Random_Value_2).Value = 1
Range("Schedule_Array").Cells(Random_Value_2, Random_Value_1).Value = 1
Schedule_Array(Schedule_Array_Size, 1) = Random_Value_1
Schedule_Array(Schedule_Array_Size, 2) = Random_Value_2
Worksheets("Sch_Gen").Cells((Schedule_Array_Size + 11), 12).Value =
Random_Value_1
Worksheets("Sch_Gen").Cells((Schedule_Array_Size + 11), 13).Value =
Random_Value_2

' Populate M1 with zeros
For i = 1 To 8
    For j = 1 To 8
        M1_Array(1, i, j) = 0
    Next
Next

' Fill in the non zero values
For i = 1 To Schedule_Array_Size
    M1_Array(1, Schedule_Array(i, 1), Schedule_Array(i, 2)) = 1
    ' Put in reciprocal
    M1_Array(1, Schedule_Array(i, 2), Schedule_Array(i, 1)) = 1
```

```
Next

' Fill in the M1_Array > 2 with all zeros
' Fill all path matrices
For n_matrix = 2 To Schedule_Array_Size ' power to calculate
  For i = 1 To 8 ' row of result màtrix
    For j = 1 To 8 ' column of result matrix
      M1_Array(n_matrix, i, j) = 0
      For m = 1 To 8 ' indexes through

          temp = M1_Array(1, i, m) * M1_Array((n_matrix - 1), m, j)
          M1_Array(n_matrix, i, j) = M1_Array(n_matrix, i, j) + temp

      Next
    Next
  Next
Next

' Populate path list array
  ' Only look at upper half of matrices

  Erase path_list
  path_list_row = 1 'initialize path_list_row to 1
  For i = 1 To 7 ' which matrix
    For j = 1 To 8 ' row
      For m = 1 To 8 ' column
        If (M1_Array(i, j, m) = 1 And (m > j)) Then
          in_list = False
          For q = 1 To path_list_length Step 1 ' Is compariosn i list?
            If ((path_list(q, 1) = j) And _
              (path_list(q, 2) = m)) Then
                in_list = True ' already in path list
                q = path_list_length ' exit for loop early
            End If
          Next
          If in_list = False Then
            path_list(path_list_row, 1) = j ' row
            path_list(path_list_row, 2) = m ' column
            path_list(path_list_row, 3) = i ' length of path
            path_list_length = path_list_row
            path_list_row = path_list_row + 1
          End If
        End If
      Next
    Next
  Next

' put path list on the spreadsheet 14, 16

For i = 1 To path_list_length
  Worksheets("Sch_Gen").Cells(((13 + i) + 3), 16).Value = path_list(i, 1)
  Worksheets("Sch_Gen").Cells(((13 + i) + 3), 17).Value = path_list(i, 2)
  Worksheets("Sch_Gen").Cells(((13 + i) + 3), 18).Value = path_list(i, 3)
Next

' Determine degree of all vertices using second power of M1_Array
```

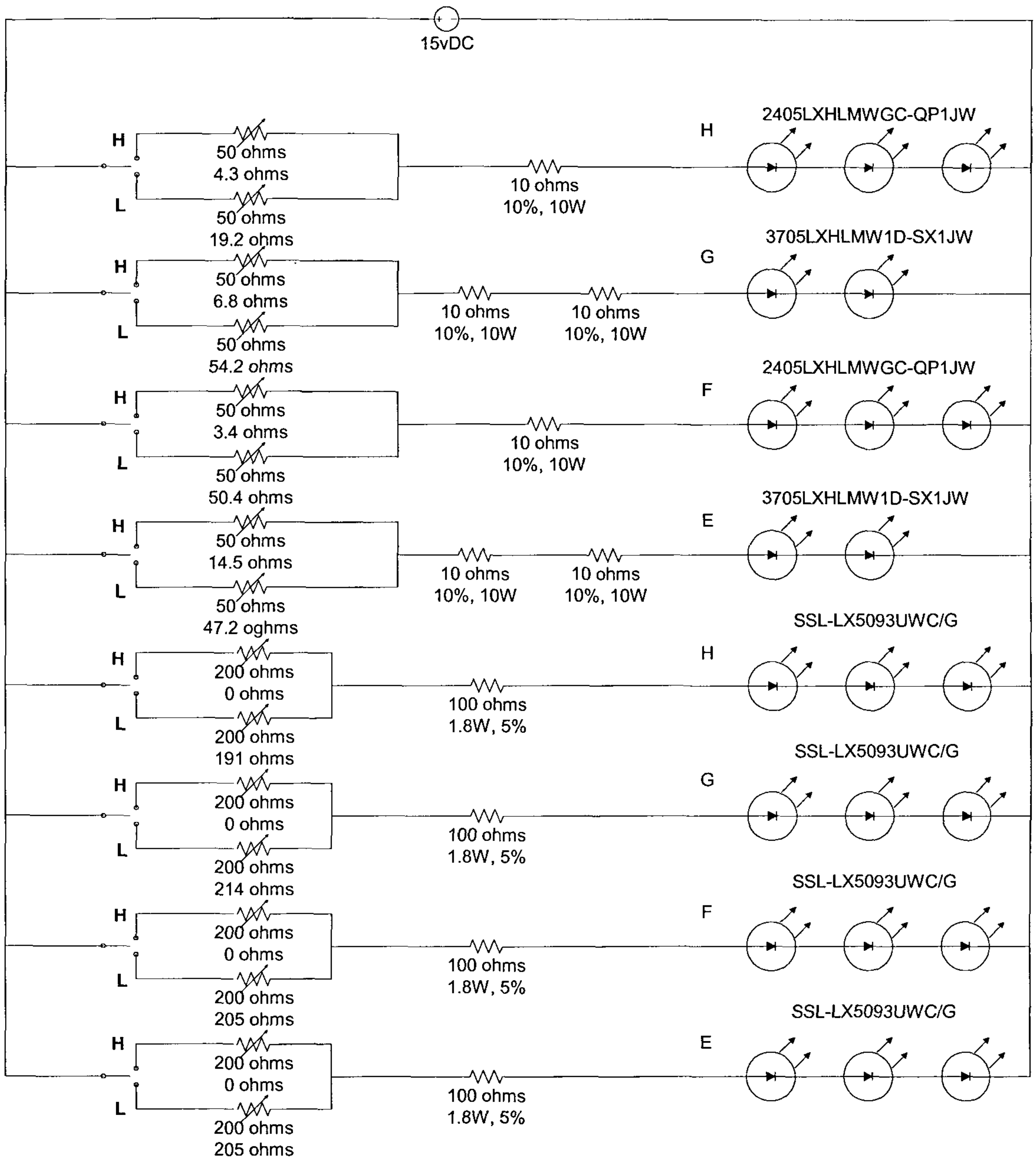
```
    For i = 1 To 8
        vertex_degrees(i, 1) = i                    ' vertex
        vertex_degrees(i, 2) = M1_Array(2, i, i)    ' degree of vertex
        Worksheets("Sch_Gen").Cells((i + 3), 16).Value = vertex_degrees(i, 1)
        Worksheets("Sch_Gen").Cells((i + 3), 17).Value = vertex_degrees(i, 2)
    Next

Next

End Sub
```

Appendix 3

Electrical Schematic of LED Lighting Models


15vDC
H
L
50 ohms
4.3 ohms
50 ohms
19.2 ohms
10 ohms
10%, 10W
H
2405LXHLMWGC-QP1JW
H
L
50 ohms
6.8 ohms
50 ohms
54.2 ohms
10 ohms
10%, 10W
10 ohms
10%, 10W
G
3705LXHLMW1D-SX1JW
H
L
50 ohms
3.4 ohms
50 ohms
50.4 ohms
10 ohms
10%, 10W
F
2405LXHLMWGC-QP1JW
H
L
50 ohms
14.5 ohms
50 ohms
47.2 oghms
10 ohms
10%, 10W
10 ohms
10%, 10W
E
3705LXHLMW1D-SX1JW
H
L
200 ohms
0 ohms
200 ohms
191 ohms
100 ohms
1.8W, 5%
H
SSL-LX5093UWC/G
H
L
200 ohms
0 ohms
200 ohms
214 ohms
100 ohms
1.8W, 5%
G
SSL-LX5093UWC/G
H
L
200 ohms
0 ohms
200 ohms
205 ohms
100 ohms
1.8W, 5%
F
SSL-LX5093UWC/G
H
L
200 ohms
0 ohms
200 ohms
205 ohms
100 ohms
1.8W, 5%
E
SSL-LX5093UWC/G

Appendix 4

Survey Materials

Survey Instructions:

You will be shown several pairs of refrigerator door lighting options and asked to state your preference of one over the other by using the scale you have been provided. Using the options 'A' and 'B', the scale allows you to state your preference with the following verbal statements.

'A' is absolutely preferred over 'B'

'A' is very strongly preferred over 'B'

'A' is strongly preferred over 'B'

'A' is slightly preferred over 'B'

equal or no preference

'B' is slightly preferred over 'A'

'B' is strongly preferred over 'A'

'B' is very strongly preferred over 'A'

'B' is absolutely preferred over 'A'

For example, given the colors 'RED' and 'BLUE' as options 'A' and 'B' and asked to state your preference of one color over the other, you may say that 'RED' is strongly preferred over 'BLUE'.

You may also state that your preference of one option over the other is in-between two of these statements. For example, you may Slightly Prefer to Strongly Prefer option 'B' over 'A'.

Additional comments concerning your preferences are welcome.

If the options are on the same door, you may ask to see the options as many times as needed to determine your preference.

Do you have any questions?

Administered questions:

Use this question for lighting options shown on different doors:

"Which lighting option do you prefer? Door X or Door X?"

Use this question when the lighting options are shown on the same door:

"The following two lighting options will be shown on the same door; Door X. *Do you prefer lighting option 1 or lighting option 2?*" (the last part of the question, shown in italics, is read twice as the lighting options are toggled on and off)

After the respondent gives an answer, repeat the answer back to the respondent in the form option X is "XXX" preferred over option X to verify that the respondent's intentions are recorded properly.

Rating scale handout:

	'A' over 'B'					'B' over 'A'				
Option 'A'	Absolutely Preferred	Very Strongly Preferred	Strongly Preferred	Slightly Preferred	Equal or No Preference	Slightly Preferred	Strongly Preferred	Very Strongly Preferred	Absolutely Preferred	Option 'B'

Example Data Collection Form/ Survey

1	5	Extreme		Very Strong		Strong		Moderate		Equal		Moderate		Strong		Very Strong		Extreme	8
Door	E1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	G2
Target (Accent)	high																		high
Background	low																		high

2	4	Extreme		Very Strong		Strong		Moderate		Equal		Moderate		Strong		Very Strong		Extreme	6
Door	H2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	E2
Target (Accent)	off																		off
Background	high																		high

3	4	Extreme		Very Strong		Strong		Moderate		Equal		Moderate		Strong		Very Strong		Extreme	8
Door	H2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	G2
Target (Accent)	off																		high
Background	high																		high

4	3	Extreme		Very Strong		Strong		Moderate		Equal		Moderate		Strong		Very Strong		Extreme	6
Door	H1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	E2
Target (Accent)	high																		off
Background	low																		high

5	5	Extreme		Very Strong		Strong		Moderate		Equal		Moderate		Strong		Very Strong		Extreme	7
Door	E1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	G1
Target (Accent)	high																		off
Background	low																		low

6	2	Extreme		Very Strong		Strong		Moderate		Equal		Moderate		Strong		Very Strong		Extreme	7
Door	F2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	G1
Target (Accent)	high																		off
Background	high																		low

7	1	Extreme		Very Strong		Strong		Moderate		Equal		Moderate		Strong		Very Strong		Extreme	2
Door	F1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	F2
Target (Accent)	off																		high
Background	low																		high

8	1	Extreme		Very Strong		Strong		Moderate		Equal		Moderate		Strong		Very Strong		Extreme	5
Door	F1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	E1
Target (Accent)	off																		high
Background	low																		low

9	2	Extreme		Very Strong		Strong		Moderate		Equal		Moderate		Strong		Very Strong		Extreme	5
Door	high	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	high
Target (Accent)	high																		low
Background	high																		high

Appendix 5

Screen shots of Excel spreadsheet schedules and calculations

Figure 46 Schedule for test 1.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0	1			1			
2	1	0			1		1	
3			0			1		
4				0		1		1
5	1	1			0		1	1
6			1	1		0		
7		1			1		0	
8				1	1			0

generate schedule

Unlock

vertex	degree
1	1
2	2
3	1
4	2
5	2
6	2
7	2
8	2

Comparison

1	5	8
2	4	6
3	4	8
4	3	6
5	5	7
6	2	7
7	1	2

Random Redundant Comparisons

8	1	5
9	2	5

#	row	column	length
1	1	2	1
2	2	7	1
3	3	6	1
4	4	6	1
5	4	8	1
6	5	7	1
7	5	8	1
8	1	7	2
9	2	5	2
10	3	4	2
11	4	5	2
12	6	8	2
13	7	8	2
14	1	5	3
15	2	8	3
16	3	8	3
17	4	7	3
18	5	6	3
19	1	8	4
20	2	4	4
21	3	5	4
22	6	7	4
23	1	4	5
24	2	6	5
25	3	7	5
26	1	6	6
27	2	3	6
28	1	3	7

Source: Product work of the author.

Figure 47 Calculations part 'a' for test 1.

Test 1 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	1	0	0	1	0	0	0
2	1	0	0	0	1	0	1	0
3	0	0	0	0	0	1	0	0
4	0	0	0	0	0	1	0	1
5	1	1	0	0	0	0	1	1
6	0	0	1	1	0	0	0	0
7	0	1	0	0	1	0	0	0
8	0	0	0	1	1	0	0	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.20	0.000	0.000	0.200	0.000	0.000	0.000	5
2	5.000	1	0.000	0.000	0.200	0.000	0.200	0.000	4
3	0.000	0.000	1	0.000	0.000	0.200	0.000	0.000	6
4	0.000	0.000	0.000	1	0.000	0.200	0.000	0.200	5
5	5.000	5.000	0.000	0.000	1	0.000	3.000	0.333	3
6	0.000	0.000	5.000	5.000	0.000	1	0.000	0.000	5
7	0.000	5.000	0.000	0.000	0.333	0.000	1	0.000	5
8	0.000	0.000	0.000	5.000	3.000	0.000	0.000	1	5

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	6	0.200	0.000	0.000	0.200	0.000	0.000	0.000
2	5.000	5	0.000	0.000	0.200	0.000	0.200	0
3	0.000	0.000	7	0.000	0.000	0.200	0.000	0.000
4	0.000	0.000	0.000	6	0.000	0.200	0.000	0.200
5	5.000	5.000	0.000	0.000	4	0.000	3.000	0.333
6	0.000	0.000	5.000	5.000	0.000	6	0.000	0.000
7	0.000	5.000	0.000	0.000	0.333	0.000	6	0.000
8	0.000	0.000	0.000	5.000	3.000	0.000	0.000	6
column sums	16.000	15.200	12.000	16.000	7.733	6.400	9.200	6.533

Converge Matrix

Source: Product work of the author.

Figure 48 Calculations part 'b' for test 1.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0234
2	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0565
3	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0180
4	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0411
5	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.2215
6	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.1242
7	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.1497
8	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.3656
									##############	

stop delta	0.000005	eigenvalue	8.380581704
delta	0.000005	C.I.	0.054369
power	60	C.R.	0.038835

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.200	1.299	0.568	0.200	0.188	0.156	0.064
2	5.000	1.000	3.142	1.374	0.200	0.455	0.200	0.155
3	0.770	0.318	1.000	0.437	0.081	0.200	0.120	0.049
4	1.761	0.728	2.288	1.000	0.186	0.200	0.275	0.200
5	5.000	5.000	12.316	5.384	1.000	1.784	3.000	0.333
6	5.316	2.197	5.000	5.000	0.561	1.000	0.829	0.340
7	6.409	5.000	8.324	3.639	0.333	1.206	1.000	0.410
8	15.650	6.469	20.325	5.000	3.000	2.944	2.442	1.000
	40.908	20.913	53.693	22.401	5.561	7.976	8.022	2.550

rnd CI	0.2295	stop delta	0.000005	eigenvalue	8.380600524
		delta	0.000000	C.I.	0.054371503
		power	7	C.R.	0.236912869

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	372348.178	155343.954	435197.031	201415.825	41021.399	64876.372	58207.315	22904.636	1351314.709	0.023
2	900800.873	375818.114	#######	487286.596	99247.213	156956.596	140825.823	55413.028	3269227.812	0.057
3	286722.943	119618.553	335120.232	155098.216	31589.115	49957.546	44822.103	17637.563	1040566.271	0.018
4	655863.288	273625.062	766577.795	354782.673	72259.140	114276.541	102530.489	40345.184	2380260.172	0.041
5	3531163.811	1473115.971	#######	#######	389028.743	615238.991	551978.737	217213.504	12814900.988	0.222
6	#######	825870.913	#######	#######	218099.601	344918.380	309462.560	121773.673	7184301.164	0.124
7	#######	995616.202	#######	#######	262937.305	415819.009	373074.443	146805.404	8661072.145	0.150
8	#######	2431191.036	#######	#######	642009.639	#######	910971.316	358468.982	21148719.446	0.366
									57850362.705	

Source: Product work of the author.

Figure 49 Schedule for test 2.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0	1		1			1	
2	1	0						1
3			0		1	1		
4	1			0		1		
5			1		0		1	
6			1	1		0		1
7	1				1		0	
8		1				1		0

generate schedule

Unlock

vertex	degree
1	2
2	2
3	2
4	1
5	2
6	2
7	1
8	2

Comparison

1	2	8
2	6	8
3	3	6
4	3	5
5	1	2
6	5	7
7	1	4

Random Redundant Comparisons

8	1	7
9	4	6

#	row	column	length
1	1	2	1
2	1	4	1
3	2	8	1
4	3	5	1
5	3	6	1
6	5	7	1
7	6	8	1
8	1	8	2
9	2	4	2
10	2	6	2
11	3	7	2
12	3	8	2
13	5	6	2
14	1	6	3
15	2	3	3
16	4	8	3
17	5	8	3
18	6	7	3
19	1	3	4
20	2	5	4
21	4	6	4
22	7	8	4
23	1	5	5
24	2	7	5
25	3	4	5
26	1	7	6
27	4	5	6
28	4	7	7

Source: Product work of the author.

Figure 50 Calculations part 'a' for test 2.

Test 2 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	1	0	1	0	0	1	0
2	1	0	0	0	0	0	0	1
3	0	0	0	0	1	1	0	0
4	1	0	0	0	0	1	0	0
5	0	0	1	0	0	0	1	0
6	0	0	1	1	0	0	0	1
7	1	0	0	0	1	0	0	0
8	0	1	0	0	0	1	0	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.20	0.000	3.000	0.000	0.000	0.200	0.000	4
2	5.000	1	0.000	0.000	0.000	0.000	0.000	0.143	5
3	0.000	0.000	1	0.000	0.333	0.333	0.000	0.000	5
4	0.333	0.000	0	1	0.000	0.200	0.000	0.000	5
5	0.000	0.000	3.000	0.000	1	0.000	5.000	0.000	5
6	0.000	0.000	3.000	5.000	0.000	1	0.000	0.200	4
7	5.000	0.000	0.000	0.000	0.200	0.000	1	0.000	5
8	0.000	7.000	0.000	0.000	0.000	5.000	0.000	1	5

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	5	0.2	0	3	0	0	0.2	0
2	5.000	6	0	0	0	0	0	0.1429
3	0.000	0.000	6	0	0.3333	0.3333	0	0
4	0.333	0.000	0.000	6	0	0.2	0	0
5	0.000	0.000	3.000	0.000	6	0	5	0
6	0.000	0.000	3.000	5.000	0.000	5	0	0.2
7	5.000	0.000	0.000	0.000	0.200	0.000	6	0
8	0.000	7.000	0.000	0.000	0.000	5.000	0.000	6
column sums	15.333	13.200	12.000	14.000	6.533	10.533	11.200	6.343

Converge Matrix

Source: Product work of the author.

Figure 51 Calculations part 'b' for test 2.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	########	########	########	########	########	########	########	########	##############	0.0199
2	########	########	########	########	########	########	########	########	##############	0.0763
3	########	########	########	########	########	########	########	########	##############	0.0488
4	########	########	########	########	########	########	########	########	##############	0.0115
5	########	########	########	########	########	########	########	########	##############	0.2249
6	########	########	########	########	########	########	########	########	##############	0.0928
7	########	########	########	########	########	########	########	########	##############	0.0673
8	########	########	########	########	########	########	########	########	##############	0.4585
									##############	

stop delta	0.000005	eigenvalue	8.167872175
delta	0.000003	C.I.	0.023981739
power	26	C.R.	0.017129814

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.0000	0.2000	0.4078	3.0000	0.0885	0.2146	0.2000	0.0434
2	5.0000	1.0000	1.5623	6.6405	0.3392	0.8222	1.1339	0.1429
3	2.4520	0.6401	1.0000	4.2504	0.3333	0.3333	0.7258	0.1065
4	0.3333	0.1506	0.2353	1.0000	0.0511	0.2000	0.1708	0.0251
5	11.2932	2.9480	3.0000	19.5766	1.0000	2.4239	5.0000	0.4906
6	4.6591	1.2162	3.0000	5.0000	0.4126	1.0000	1.3791	0.2000
7	5.0000	0.8819	1.3778	5.8564	0.2000	0.7251	1.0000	0.1468
8	23.0181	7.0000	9.3876	39.9014	2.0382	5.0000	6.8133	1.0000
	52.756	14.037	19.971	85.225	4.463	10.719	16.423	2.155

rnd CI	0.1146	stop delta	0.000005	eigenvalue	8.169639213
		delta	0.000002	C.I.	0.024234173
		power	7	C.R.	0.21153961

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	######	######	124919.451	######	27031.625	######	######	12684.602	1246770.606	0.0199
2	######	######	######	######	103401.218	######	######	48521.073	4769141.860	0.0763
3	######	######	######	######	66206.171	161718.111	######	31067.285	3053607.445	0.0488
4	179411.664	######	72224.410	######	15628.837	38175.613	51979.989	7333.835	720843.582	0.0115
5	######	######	######	######	######	######	######	######	14029225.063	0.2243
6	######	######	######	######	######	######	######	59163.879	5815225.441	0.0930
7	######	######	######	######	91036.646	######	######	42718.991	4198854.224	0.0671
8	######	######	######	######	######	######	######	######	28702953.121	0.4590
									62536621.341	

Source: Work of the author

Figure 52 Schedule for test 3.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0							1
2		0		1				
3			0	1			1	
4		1	1	0				
5					0	1	1	
6					1	0	1	1
7			1		1	1	0	1
8	1					1	1	0

generate schedule

Unlock

Comparison

1	5	7
2	3	4
3	3	7
4	5	6
5	1	8
6	2	4
7	6	8

Random Redundant Comparisons

8	7	8
9	6	7

vertex	degree
1	1
2	1
3	2
4	2
5	2
6	2
7	2
8	2

#	row	column	length
1	1	8	1
2	2	4	1
3	3	4	1
4	3	7	1
5	5	6	1
6	5	7	1
7	6	8	1
8	1	6	2
9	2	3	2
10	3	5	2
11	4	7	2
12	5	8	2
13	6	7	2
14	1	5	3
15	2	7	3
16	3	6	3
17	4	5	3
18	7	8	3
19	1	7	4
20	2	5	4
21	3	8	4
22	4	6	4
23	1	3	5
24	2	6	5
25	4	8	5
26	1	4	6
27	2	8	6
28	1	2	7

Source: Product work of the author.

Figure 53 Calculations part 'a' for test 3.

Test 3 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	1
2	0	0	0	1	0	0	0	0
3	0	0	0	1	0	0	1	0
4	0	1	1	0	0	0	0	0
5	0	0	0	0	0	1	1	0
6	0	0	0	0	1	0	1	1
7	0	0	1	0	1	1	0	1
8	1	0	0	0	0	1	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.000	0.000	0.000	0.000	0.200	6
2	0.000	1	0.000	3.000	0.000	0.000	0.000	0.000	6
3	0.000	0.000	1	3.000	0.000	0.000	3.000	0.000	5
4	0.000	0.333	0.333	1	0.000	0.000	0.000	0.000	5
5	0.000	0.000	0.000	0.000	1	0.200	0.333	0.000	5
6	0.000	0.000	0.000	0.000	5.000	1	0.000	3.000	5
7	0.000	0.000	0.333	0.000	3.000	0.000	1	0.333	4
8	5.000	0.000	0.000	0.000	0.000	0.333	3.000	1	4

Problem with data collection sheet so this value wasn't requested

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	7	0.000	0.000	0.000	0.000	0.000	0.000	0.200
2	0.000	7	0.000	3.000	0.000	0.000	0.000	0
3	0.000	0.000	6	3.000	0.000	0.000	3.000	0.000
4	0.000	0.333	0.333	6	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	6	0.200	0.333	0.000
6	0.000	0.000	0.000	0.000	5.000	6	0.000	3.000
7	0.000	0.000	0.333	0.000	3.000	0.000	5	0.333
8	5.000	0.000	0.000	0.000	0.000	0.333	3.000	5

Converge Matrix

column sums	12.000	7.333	6.667	12.000	14.000	6.533	11.333	8.533

Source: Product work of the author.

Figure 54 Calculations part 'b' for test 3.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0281
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1379
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1835
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0509
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0431
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.3234
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0780
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1552
									###############	

stop delta	0.000005	eigenvalue	8.105831509
delta	0.000005	C.I.	0.015119
power	123	C.R.	0.010799

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.204	0.153	0.552	0.652	0.087	0.360	0.200
2	4.909	1.000	0.751	3.000	3.201	0.426	1.768	0.888
3	6.534	1.331	1.000	3.000	4.261	0.567	3.000	1.182
4	1.811	0.333	0.333	1.000	1.181	0.157	0.652	0.328
5	1.533	0.312	0.235	0.847	1.000	0.200	0.333	0.277
6	11.517	2.346	1.763	6.360	5.000	1.000	4.149	3.000
7	2.776	0.565	0.333	1.533	3.000	0.241	1.000	0.333
8	5.000	1.126	0.846	3.053	3.605	0.333	3.000	1.000
	35.080	7.218	5.414	19.345	21.900	3.012	14.264	7.208

		stop delta	0.000005	eigenvalue	8.105849558
RI	0.09828	delta	0.000002	C.I.	0.015121365
		power	7	C.R.	0.153856339

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	280220.278	57098.998	43282.048	155427.618	192848.996	25267.095	106967.847	52602.512	913715.392	0.0281
2	#######	280337.203	212500.553	763098.219	946824.720	124053.084	525176.770	258260.961	4486040.435	0.1379
3	#######	373089.365	282808.329	#######	#######	165097.242	698936.149	343708.993	5970288.371	0.1835
4	507480.412	103406.589	78384.019	281480.242	349250.547	45758.844	193719.339	95263.430	1654743.422	0.0509
5	429697.925	87557.271	66369.958	238337.238	295720.252	38745.274	164027.754	80662.233	1401117.906	0.0431
6	#######	657622.335	498489.255	#######	#######	291006.965	#######	605834.961	10523471.986	0.3234
7	777849.823	158498.327	120144.563	431443.955	535319.534	70137.699	296926.838	146016.878	2536337.618	0.0780
8	#######	315624.304	239248.798	859152.304	1066005.116	139668.211	591282.277	290769.242	5050715.028	0.1552
									32536430.157	

Source: Product work of the author.

Figure 55 Schedule for test 4.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0	1	1					
2	1	0			1			
3	1		0				1	
4				0		1	1	
5		1			0			1
6				1		0		1
7			1	1			0	1
8					1	1	1	0

generate s chedule

Unlock

vertex	degree
1	2
2	2
3	1
4	2
5	2
6	2
7	1
8	2

Comparison

1	6	8
2	4	6
3	5	8
4	4	7
5	1	3
6	2	5
7	1	2

Random Redundant Comparisons

8	3	7
9	7	8

#	row	column	length
1	1	2	1
2	1	3	1
3	2	5	1
4	4	6	1
5	4	7	1
6	5	8	1
7	6	8	1
8	1	5	2
9	2	3	2
10	2	8	2
11	4	8	2
12	5	6	2
13	6	7	2
14	1	8	3
15	2	6	3
16	3	5	3
17	4	5	3
18	7	8	3
19	1	6	4
20	2	4	4
21	3	8	4
22	5	7	4
23	1	4	5
24	2	7	5
25	3	6	5
26	1	7	6
27	3	4	6
28	3	7	7

Source: Product work of the author.

Figure 56 Calculations part 'a' of test 4.

Test 4 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	0	0
2	1	0	0	0	1	0	0	0
3	1	0	0	0	0	0	1	0
4	0	0	0	0	0	1	1	0
5	0	1	0	0	0	0	0	1
6	0	0	0	1	0	0	0	1
7	0	0	1	1	0	0	0	1
8	0	0	0	0	1	1	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.33	3.000	0.000	0.000	0.000	0.000	0.000	5
2	3.000	1	0.000	0.000	0.200	0.000	0.000	0.000	5
3	0.333	0.000	1	0.000	0.000	0.000	0.200	0.000	5
4	0.000	0.000	0.000	1	0.000	0.200	0.333	0.000	5
5	0.000	5.000	0.000	0.000	1	0.000	0.000	1.000	5
6	0.000	0.000	0.000	5.000	0.000	1	0.000	3.000	5
7	0.000	0.000	5.000	3.000	0.000	0.000	1	0.000	5
8	0.000	0.000	0.000	0.000	1.000	0.333	0.000	1	5

Error on data collection sheet so this value not collected

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	6	0.333	3.000	0.000	0.000	0.000	0.000	0.000
2	3.000	6	0.000	0.000	0.200	0.000	0.000	0
3	0.333	0.000	6	0.000	0.000	0.000	0.200	0.000
4	0.000	0.000	0.000	6	0.000	0.200	0.333	0.000
5	0.000	5.000	0.000	0.000	6	0.000	0.000	1.000
6	0.000	0.000	0.000	5.000	0.000	6	0.000	3.000
7	0.000	0.000	5.000	3.000	0.000	0.000	6	0.000
8	0.000	0.000	0.000	0.000	1.000	0.333	0.000	6
column sums	9.333	11.333	14.000	14.000	7.200	6.533	6.533	10.000

Converge Matrix

Source: Product work of the author.

Figure 57 Calculations part 'b' for test 4.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0309
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0653
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0146
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0487
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.2304
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.3445
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1032
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1626
									###############	

stop delta	0.000005	eigenvalue	8.122625148
delta	0.000003	C.I.	0.017518
power	92	C.R.	0.012513

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.333	3.000	0.634	0.134	0.090	0.299	0.190
2	3.000	1.000	4.482	1.342	0.200	0.190	0.633	0.402
3	0.333	0.223	1.000	0.299	0.063	0.042	0.200	0.090
4	1.578	0.745	3.341	1.000	0.211	0.200	0.333	0.299
5	7.467	5.000	15.812	4.733	1.000	0.669	2.233	1.000
6	11.166	5.275	23.644	5.000	1.495	1.000	3.339	3.000
7	3.344	1.580	5.000	3.000	0.448	0.299	1.000	0.634
8	5.271	2.490	11.160	3.341	1.000	0.333	1.576	1.000
	33.159	16.647	67.439	19.349	4.552	2.823	9.615	6.615

RI 0.09828

stop delta	0.000005	eigenvalue	8.122421166
delta	0.000004	C.I.	0.017488738
power	6	C.R.	0.177943798

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	35896.645	16958.200	76009.428	22749.636	4806.674	3214.195	10734.898	6809.625	177179.299	0.0308
2	75987.478	35897.733	160899.861	48157.268	10174.946	6803.936	22724.019	14414.887	375060.126	0.0653
3	16951.561	8008.209	35894.045	10743.117	2269.864	1517.847	5069.356	3215.720	83669.718	0.0146
4	56631.555	26753.684	119914.567	35890.391	7583.131	5070.802	16935.640	10743.057	279522.826	0.0487
5	268080.358	126645.554	567646.114	169896.586	35896.668	24003.966	80169.153	50854.971	1323193.369	0.2304
6	400841.997	189364.685	848763.529	254035.081	53674.017	35891.475	119871.783	76040.060	1978482.627	0.3445
7	120018.637	56698.795	254133.285	76062.139	16070.823	10746.500	35891.455	22767.602	592389.238	0.1031
8	189214.450	89388.206	400651.823	119915.339	25336.369	16942.305	56584.492	35894.074	933927.057	0.1626
									5743424.260	

Source: Product work of the author.

Figure 58 Schedule for test 5.

generate schedule

Unlock

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0		1				1	
2		0		1				1
3	1		0	1	1			
4		1	1	0				
5			1		0		1	
6						0		1
7	1				1		0	1
8		1				1	1	0

vertex	degree
1	1
2	2
3	2
4	2
5	2
6	1
7	2
8	2

Comparison

1	2	8
2	2	4
3	5	7
4	1	7
5	3	4
6	3	5
7	6	8

Random Redundant Comparisons

8	7	8
9	1	3

#	row	column	length
1	1	7	1
2	2	4	1
3	2	8	1
4	3	4	1
5	3	5	1
6	5	7	1
7	6	8	1
8	1	5	2
9	2	3	2
10	2	6	2
11	3	7	2
12	4	5	2
13	4	8	2
14	1	3	3
15	2	5	3
16	3	8	3
17	4	6	3
18	4	7	3
19	1	4	4
20	2	7	4
21	3	6	4
22	5	8	4
23	1	2	5
24	5	6	5
25	7	8	5
26	1	8	6
27	6	7	6
28	1	6	7

Source: Product work of the author.

Figure 59 Calculations part 'a' for test 5.

Test 5 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	1	0	0	0	1	0
2	0	0	0	1	0	0	0	1
3	1	0	0	1	1	0	0	0
4	0	1	1	0	0	0	0	0
5	0	0	1	0	0	0	1	0
6	0	0	0	0	0	0	0	1
7	1	0	0	0	1	0	0	1
8	0	1	0	0	0	1	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.200	0.000	0.000	0.000	0.143	0.000	5
2	0.000	1	0.000	1.000	0.000	0.000	0.000	0.143	5
3	5.000	0.000	1	0.200	0.200	0.000	0.000	0.000	4
4	0.000	1.000	5.000	1	0.000	0.000	0.000	0.000	5
5	0.000	0.000	5.000	0.000	1	0.000	1.000	0.000	5
6	0.000	0.000	0.000	0.000	0.000	1	0.000	0.333	6
7	7.000	0.000	0.000	0.000	1.000	0.000	1	0.200	4
8	0.000	7.000	0.000	0.000	0.000	3.000	5.000	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	6	0.000	0.200	0.000	0.000	0.000	0.143	0.000
2	0.000	6	0.000	1.000	0.000	0.000	0.000	0.14286
3	5.000	0.000	5	0.200	0.200	0.000	0.000	0.000
4	0.000	1.000	5.000	6	0.000	0.000	0.000	0.000
5	0.000	0.000	5.000	0.000	6	0.000	1.000	0.000
6	0.000	0.000	0.000	0.000	0.000	7	0.000	0.333
7	7.000	0.000	0.000	0.000	1.000	0.000	5	0.200
8	0.000	7.000	0.000	0.000	0.000	3.000	5.000	5
column sums	18.000	14.000	15.200	7.200	7.200	10.000	11.143	5.676

Converge Matrix

Source: Product work of the author.

Figure 60 Calculations part 'b' for test 5.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0082
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0810
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0268
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1035
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1040
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1404
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0820
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.4541
									###############	

stop delta	0.000005	eigenvalue	8.078407116
delta	0.000005	C.I.	0.011201
power	98	C.R.	0.008001

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.101	0.200	0.079	0.079	0.059	0.143	0.018
2	9.863	1.000	3.021	1.000	0.779	0.577	0.989	0.143
3	5.000	0.331	1.000	0.200	0.200	0.191	0.327	0.059
4	12.601	1.000	5.000	1.000	0.996	0.738	1.263	0.228
5	12.656	1.283	5.000	1.004	1.000	0.741	1.000	0.229
6	17.085	1.732	5.232	1.356	1.350	1.000	1.712	0.333
7	7.000	1.012	3.055	0.792	1.000	0.584	1.000	0.200
8	55.281	7.000	16.930	4.387	4.368	3.000	5.000	1.000
	120.486	13.459	39.438	9.818	9.772	6.889	11.434	2.210

RI 0.22947	stop delta	0.000005	eigenvalue	8.078364748
	delta	0.000001	C.I.	0.011194964
	power	7	C.R.	0.048785545

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	286305.319	28284.100	87554.539	22175.776	22073.198	16121.526	28298.394	5019.981	495832.832	0.0082
2	#######	278965.973	863549.958	218719.590	217707.864	159006.549	279106.994	49512.050	4890397.394	0.0810
3	934863.936	92355.220	285889.153	72409.897	72074.957	52641.132	92401.906	16391.592	1619027.794	0.0268
4	#######	356417.231	1103303.216	279444.229	278151.613	203152.638	356597.431	63258.423	6248152.095	0.1035
5	#######	357966.720	#######	280659.085	279360.849	204035.824	358147.711	63533.433	6275315.303	0.1040
6	#######	483236.953	#######	378875.562	377123.009	275438.037	483481.236	85766.919	8471358.205	0.1404
7	#######	282188.024	873523.940	221245.791	220222.373	160843.070	282330.673	50083.912	4946881.601	0.0820
8	#######	#######	4840167.118	#######	#######	891226.110	#######	277513.297	27410504.745	0.4541
									60357469.968	

Source: Product work of the author.

Figure 61 Schedule for test 6.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0				1		1	
2		0						1
3			0	1				
4			1	0		1		
5	1				0	1	1	1
6				1	1	0		
7	1				1		0	1
8		1			1		1	0

generate schedule

Unlock

vertex	degree
1	2
2	1
3	1
4	2
5	2
6	2
7	2
8	2

Comparison

1	5	6
2	4	6
3	1	7
4	7	8
5	1	5
6	3	4
7	2	8

Random Redundant Comparisons

8	5	7
9	5	8

#	row	column	length
1	1	5	1
2	1	7	1
3	2	8	1
4	3	4	1
5	4	6	1
6	5	6	1
7	7	8	1
8	1	6	2
9	1	8	2
10	2	7	2
11	3	6	2
12	4	5	2
13	5	7	2
14	1	2	3
15	1	4	3
16	3	5	3
17	5	8	3
18	6	7	3
19	1	3	4
20	2	5	4
21	4	7	4
22	6	8	4
23	2	6	5
24	3	7	5
25	4	8	5
26	2	4	6
27	3	8	6
28	2	3	7

Source: Product work of the author.

Figure 62 Calculations part 'a' for test 6.

Test 6 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	0	1	0	1	0
2	0	0	0	0	0	0	0	1
3	0	0	0	1	0	0	0	0
4	0	0	1	0	0	1	0	0
5	1	0	0	0	0	1	1	1
6	0	0	0	1	1	0	0	0
7	1	0	0	0	1	0	0	1
8	0	1	0	0	1	0	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.000	0.200	0.000	0.200	0.000	5
2	0.000	1	0.000	0.000	0.000	0.000	0.000	0.200	6
3	0.000	0.000	1	0.200	0.000	0.000	0.000	0.000	6
4	0.000	0.000	5.000	1	0.000	0.200	0.000	0.000	5
5	5.000	0.000	0.000	0.000	1	0.200	0.200	0.333	3
6	0.000	0.000	0.000	5.000	5.000	1	0.000	0.000	5
7	5.000	0.000	0.000	0.000	5.000	0.000	1	5.000	4
8	0.000	5.000	0.000	0.000	3.000	0.000	0.200	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	6	0.000	0.000	0.000	0.200	0.000	0.200	0.000
2	0.000	7	0.000	0.000	0.000	0.000	0.000	0.2
3	0.000	0.000	7	0.200	0.000	0.000	0.000	0.000
4	0.000	0.000	5.000	6	0.000	0.200	0.000	0.000
5	5.000	0.000	0.000	0.000	4	0.200	0.200	0.333
6	0.000	0.000	0.000	5.000	5.000	6	0.000	0.000
7	5.000	0.000	0.000	0.000	5.000	0.000	5	5.000
8	0.000	5.000	0.000	0.000	3.000	0.000	0.200	5
column sums	16.000	12.000	12.000	11.200	17.200	6.400	5.600	10.533

Converge Matrix

Source: Product work of the author.

Figure 63 Calculations part 'b' for test 6.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0425
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0205
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0051
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0338
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0905
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.2666
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.4045
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1366
									###############	

stop delta	0.000005	eigenvalue	8.331458541
delta	0.000005	C.I.	0.047351
power	110	C.R.	0.033822

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	2.070	8.362	1.257	0.200	0.159	0.200	0.311
2	0.483	1.000	4.039	0.607	0.227	0.077	0.051	0.200
3	0.120	0.248	1.000	0.200	0.056	0.019	0.013	0.037
4	0.795	1.646	5.000	1.000	0.373	0.200	0.083	0.247
5	5.000	4.413	17.824	2.680	1.000	0.200	0.200	0.333
6	6.277	12.995	52.491	5.000	5.000	1.000	0.659	1.952
7	5.000	19.722	79.661	11.979	5.000	1.518	1.000	5.000
8	3.216	5.000	26.894	4.044	3.000	0.512	0.200	1.000
	21.891	47.094	195.271	26.769	14.856	3.685	2.406	9.080

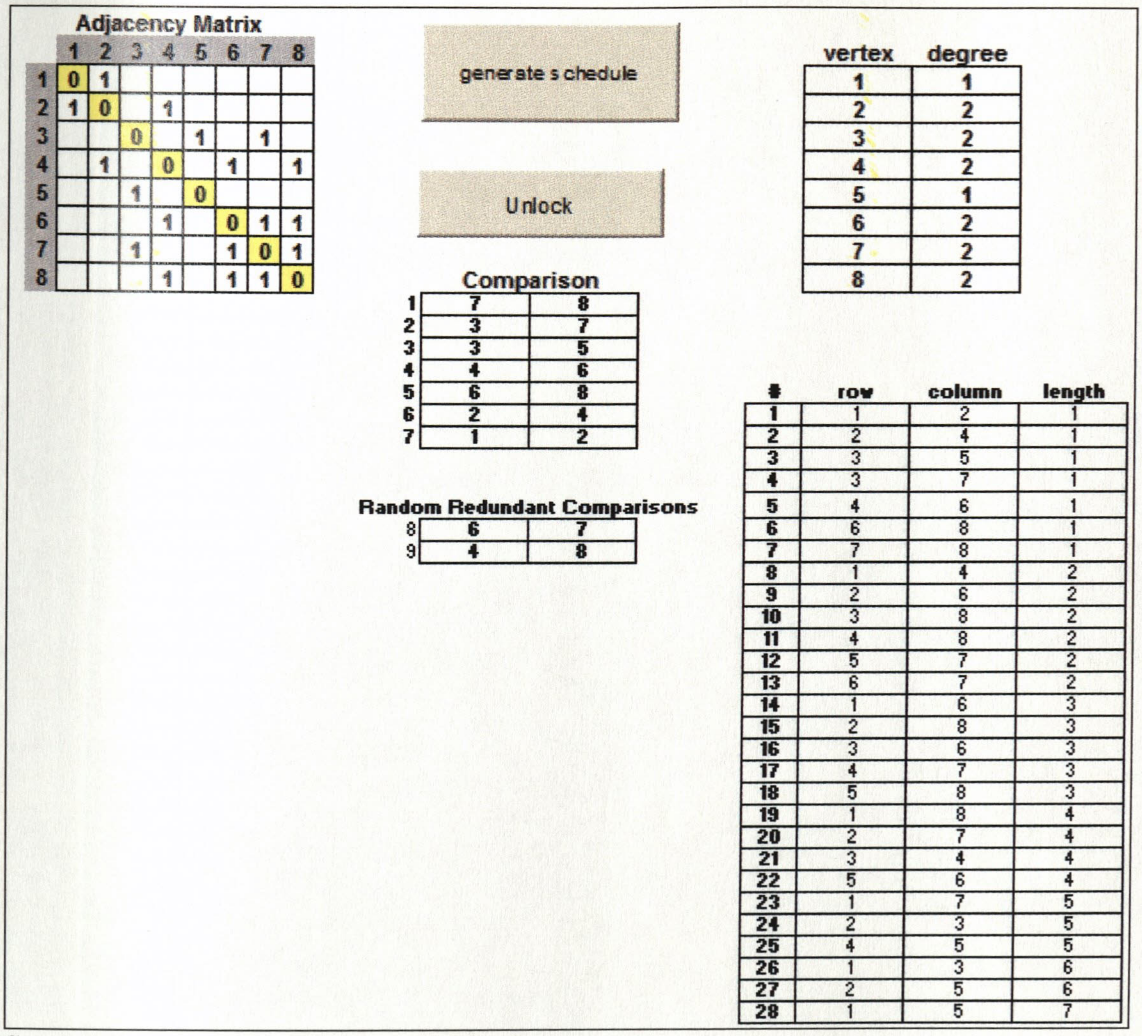
RI	0.22947	stop delta	0.000005	eigenvalue	8.331249276
		delta	0.000001	C.I.	0.047321325
		power	9	C.R.	0.206217423

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	#######	#######	#######	#######	#######	#######	#######	#######	316454087.143	0.0425
2	#######	#######	#######	#######	#######	#######	#######	#######	152860678.010	0.0205
3	3122639.114	#######	#######	#######	#######	447596.341	307607.324	912316.462	37836566.342	0.0051
4	#######	#######	#######	#######	#######	#######	#######	#######	251628457.008	0.0338
5	#######	#######	#######	#######	#######	#######	#######	#######	674527296.735	0.0905
6	#######	#######	#######	#######	#######	#######	#######	#######	1986395552.615	0.2666
7	#######	#######	#######	#######	#######	#######	#######	#######	3014706607.658	0.4045
8	#######	#######	#######	#######	#######	#######	8274411.691	#######	1017776999.675	0.1366
									7452186245.186	

Source: Product work of the author.

Figure 64 Schedule for test 7.



Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0	1						
2	1	0		1				
3			0		1		1	
4		1		0		1		1
5			1		0			
6				1		0	1	1
7			1			1	0	1
8				1		1	1	0

generate schedule

Unlock

Comparison

1	7	8
2	3	7
3	3	5
4	4	6
5	6	8
6	2	4
7	1	2

Random Redundant Comparisons

8	6	7
9	4	8

vertex	degree
1	1
2	2
3	2
4	2
5	1
6	2
7	2
8	2

#	row	column	length
1	1	2	1
2	2	4	1
3	3	5	1
4	3	7	1
5	4	6	1
6	6	8	1
7	7	8	1
8	1	4	2
9	2	6	2
10	3	8	2
11	4	8	2
12	5	7	2
13	6	7	2
14	1	6	3
15	2	8	3
16	3	6	3
17	4	7	3
18	5	8	3
19	1	8	4
20	2	7	4
21	3	4	4
22	5	6	4
23	1	7	5
24	2	3	5
25	4	5	5
26	1	3	6
27	2	5	6
28	1	5	7

Source: Product work of the author.

Figure 65 Calcuations part 'a' for test 7.

Test 7 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	1	0	0	0	0	0	0
2	1	0	0	1	0	0	0	0
3	0	0	0	0	1	0	1	0
4	0	1	0	0	0	1	0	1
5	0	0	1	0	0	0	0	0
6	0	0	0	1	0	0	1	1
7	0	0	1	0	0	1	0	1
8	0	0	0	1	0	1	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.20	0.000	0.000	0.000	0.000	0.000	0.000	6
2	5.000	1	0.000	0.200	0.000	0.000	0.000	0.000	5
3	0.000	0.000	1	0.000	0.333	0.000	0.200	0.000	5
4	0.000	5.000	0.000	1	0.000	0.200	0.000	3.000	4
5	0.000	0.000	3.000	0.000	1	0.000	0.000	0.000	6
6	0.000	0.000	0.000	5.000	0.000	1	3.000	5.000	4
7	0.000	0.000	5.000	0.000	0.000	0.333	1	7.000	4
8	0.000	0.000	0.000	0.333	0.000	0.200	0.143	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	7	0.200	0.000	0.000	0.000	0.000	0.000	0.000
2	5.000	6	0.000	0.200	0.000	0.000	0.000	0
3	0.000	0.000	6	0.000	0.333	0.000	0.200	0.000
4	0.000	5.000	0.000	5	0.000	0.200	0.000	3.000
5	0.000	0.000	3.000	0.000	7	0.000	0.000	0.000
6	0.000	0.000	0.000	5.000	0.000	5	3.000	5.000
7	0.000	0.000	5.000	0.000	0.000	0.333	5	7.000
8	0.000	0.000	0.000	0.333	0.000	0.200	0.143	5
column sums	12.000	11.200	14.000	10.533	7.333	5.733	8.343	20.000

Converge Matrix

Source: Product work of the author.

Figure 66 Calculations part 'b' for test 7.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0031
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0173
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0359
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1070
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0957
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.4670
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.2225
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0515
									###############	

stop delta	0.000005	eigenvalue	8.125752601
delta	0.000005	C.I.	0.017965
power	146	C.R.	0.012832

Resulting matrix will use Harker estimate for missing comparisons
M5

0	1	2	3	4	5	6	7	8
1	1.000	0.200	0.086	0.029	0.032	0.007	0.014	0.060
2	5.000	1.000	0.482	0.200	0.181	0.037	0.078	0.336
3	11.680	2.077	1.000	0.336	0.333	0.077	0.200	0.698
4	34.761	5.000	2.976	1.000	1.117	0.200	0.481	3.000
5	31.108	5.531	3.000	0.895	1.000	0.205	0.430	1.860
6	151.769	26.985	12.994	5.000	4.879	1.000	3.000	5.000
7	72.319	12.858	5.000	2.080	2.325	0.333	1.000	7.000
8	16.724	2.973	1.432	0.333	0.538	0.200	0.143	1.000
	324.360	56.625	26.969	9.874	10.405	2.059	5.345	18.955

RI	0.22947	stop delta	0.000005	eigenvalue	8.125639942
		delta	0.000003	C.I.	0.017948563
		power	6	C.R.	0.078216458

Calculate new eigen vector using Harker values and original values
M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	34940.032	6244.086	3004.746	1026.077	1123.012	245.963	506.514	2277.746	49368.176	0.0031
2	196517.474	35119.365	16899.953	5771.087	6316.291	1383.397	2848.846	12811.035	277667.447	0.0173
3	408164.531	72942.510	35101.000	11986.480	13118.863	2873.299	5917.018	26608.406	576712.106	0.0359
4	1214648.615	217068.271	104456.503	35670.214	39040.161	8550.680	17608.276	79182.898	1716225.617	0.1070
5	#######	194280.712	93490.728	31925.730	34941.780	7652.966	15759.866	70870.609	1536058.980	0.0957
6	#######	947753.111	456072.276	155742.593	170455.498	37332.892	76880.870	345730.124	7493314.909	0.4670
7	2527118.031	451617.980	217325.391	74212.940	81224.394	17790.067	36634.502	164742.699	3570666.005	0.2225
8	584378.950	104433.708	50255.048	17161.443	18782.587	4113.756	8471.652	38095.195	825692.339	0.0515
									16045705.580	

Source: Product work of the author.

Figure 67 Schedule for test 8.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0				1			
2		0		1		1		
3			0	1			1	1
4		1	1	0				
5	1				0	1		
6		1			1	0		1
7			1				0	1
8			1			1	1	0

generate s chedule

Unlock

vertex	degree
1	1
2	2
3	2
4	2
5	2
6	2
7	1
8	2

Comparison

1	7	8
2	3	8
3	5	6
4	2	4
5	3	4
6	2	6
7	1	5

Random Redundant Comparisons

8	3	7
9	6	8

#	row	column	length
1	1	5	1
2	2	4	1
3	2	6	1
4	3	4	1
5	3	8	1
6	5	6	1
7	7	8	1
8	1	6	2
9	2	3	2
10	2	5	2
11	3	7	2
12	4	6	2
13	4	8	2
14	1	2	3
15	2	8	3
16	3	6	3
17	4	5	3
18	4	7	3
19	1	4	4
20	2	7	4
21	3	5	4
22	6	8	4
23	1	3	5
24	5	8	5
25	6	7	5
26	1	8	6
27	5	7	6
28	1	7	7

Source: Product work of the author.

Figure 68 Calculations part 'a' for test 8.

Test 8 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	0	1	0	0	0
2	0	0	0	1	0	1	0	0
3	0	0	0	1	0	0	1	1
4	0	1	1	0	0	0	0	0
5	1	0	0	0	0	1	0	0
6	0	1	0	0	1	0	0	1
7	0	0	1	0	0	0	0	1
8	0	0	1	0	0	1	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.000	3.000	0.000	0.000	0.000	6
2	0.000	1	0.000	0.333	0.000	3.000	0.000	0.000	5
3	0.000	0.000	1	0.143	0.000	0.000	0.200	1.000	4
4	0.000	3.000	7.000	1	0.000	0.000	0.000	0.000	5
5	0.333	0.000	0.000	0.000	1	0.333	0.000	0.000	5
6	0.000	0.333	0.000	0.000	3.000	1	0.000	5.000	4
7	0.000	0.000	5.000	0.000	0.000	0.000	1	3.000	5
8	0.000	0.000	1.000	0.000	0.000	0.200	0.333	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	7	0.000	0.000	0.000	3.000	0.000	0.000	0.000
2	0.000	6	0.000	0.333	0.000	3.000	0.000	0
3	0.000	0.000	5	0.143	0.000	0.000	0.200	1.000
4	0.000	3.000	7.000	6	0.000	0.000	0.000	0.000
5	0.333	0.000	0.000	0.000	6	0.333	0.000	0.000
6	0.000	0.333	0.000	0.000	3.000	5	0.000	5.000
7	0.000	0.000	5.000	0.000	0.000	0.000	6	3.000
8	0.000	0.000	1.000	0.000	0.000	0.200	0.333	5
column sums	7.333	9.333	18.000	6.476	12.000	8.533	6.533	14.000

Converge Matrix

Source: Product work of the author.

Figure 69 Calculations part 'b' for test 8.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0752
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.2006
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0365
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.4014
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0274
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0980
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1293
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0316
									###############	

stop delta	0.000005	eigenvalue	8.13250615
delta	0.000000	C.I.	0.018929
power	36	C.R.	0.013521

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.375	2.059	0.187	3.000	0.768	0.582	2.383
2	2.667	1.000	5.492	0.333	7.327	3.000	1.552	6.355
3	0.486	0.182	1.000	0.143	1.334	0.373	0.200	1.000
4	5.337	3.000	7.000	1.000	14.661	4.097	3.106	12.717
5	0.333	0.136	0.750	0.068	1.000	0.333	0.212	0.867
6	1.303	0.333	2.683	0.244	3.000	1.000	0.758	5.000
7	1.719	0.644	5.000	0.322	4.721	1.319	1.000	3.000
8	0.420	0.157	1.000	0.079	1.153	0.200	0.333	1.000
	13.264	5.829	24.983	2.377	36.196	11.089	7.743	32.323

RI 0.22947

stop delta	0.000005	eigenvalue	8.129109541
delta	0.000001	C.I.	0.01844422
power	7	C.R.	0.080376437

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	284292.592	110156.100	606327.781	55432.689	781675.061	228005.857	168690.741	703162.035	2937742.857	0.0748
2	762056.542	295277.382	#######	148589.339	#######	611177.726	452181.674	1884851.110	7874725.618	0.2006
3	138914.374	53825.762	296270.958	27086.162	381951.214	111410.919	82427.646	343587.305	1435474.340	0.0366
4	#######	591118.467	#######	297462.364	4194617.137	#######	905226.737	#######	15764489.151	0.4016
5	103712.014	40185.749	221192.812	20222.250	285160.772	83178.196	61539.618	256518.645	1071710.056	0.0273
6	372257.630	144240.319	793936.158	72584.532	#######	298554.789	220886.559	920732.193	3846731.076	0.0980
7	491828.426	190570.950	#######	95898.987	#######	394451.958	291836.305	#######	5082318.201	0.1295
8	120078.165	46527.221	256097.839	23413.400	330160.236	96304.054	71250.788	296998.243	1240829.947	0.0316
									39254021.245	

Source: Product work of the author.

Figure 70 Schedule for test 9.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0							1
2		0	1		1		1	
3		1	0					
4				0	1	1		
5		1		1	0			1
6				1		0	1	
7		1				1	0	1
8	1				1		1	0

generate schedule

Unlock

vertex	degree
1	1
2	2
3	1
4	2
5	2
6	2
7	2
8	2

Comparison

1	7	8
2	6	7
3	2	5
4	4	6
5	2	3
6	4	5
7	1	8

Random Redundant Comparisons

8	2	7
9	5	8

#	row	column	length
1	1	8	1
2	2	3	1
3	2	5	1
4	4	5	1
5	4	6	1
6	6	7	1
7	7	8	1
8	1	7	2
9	2	4	2
10	3	5	2
11	4	7	2
12	5	6	2
13	6	8	2
14	1	6	3
15	2	6	3
16	3	4	3
17	4	8	3
18	5	7	3
19	1	4	4
20	2	7	4
21	3	6	4
22	5	8	4
23	1	5	5
24	2	8	5
25	3	7	5
26	1	2	6
27	3	8	6
28	1	3	7

Source: Product work of the author.

Figure 71 Calculations part 'a' for test 9.

Test 9 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	1
2	0	0	1	0	1	0	1	0
3	0	1	0	0	0	0	0	0
4	0	0	0	0	1	1	0	0
5	0	1	0	1	0	0	0	1
6	0	0	0	1	0	0	1	0
7	0	1	0	0	0	1	0	1
8	1	0	0	0	1	0	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.000	0.000	0.000	0.000	0.111	6
2	0.000	1	1.000	0.000	5.000	0.000	3.000	0.000	4
3	0.000	1.000	1	0.000	0.000	0.000	0.000	0.000	6
4	0.000	0.000	0.000	1	0.200	0.111	0.000	0.000	5
5	0.000	0.200	0.000	5.000	1	0.000	0.000	0.111	4
6	0.000	0.000	0.000	9.000	0.000	1	3.000	0.000	5
7	0.000	0.333	0.000	0.000	0.000	0.333	1	0.111	4
8	9.000	0.000	0.000	0.000	9.000	0.000	9.000	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	7	0.000	0.000	0.000	0.000	0.000	0.000	0.111
2	0.000	5	1.000	0.000	5.000	0.000	3.000	0
3	0.000	1.000	7	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	6	0.200	0.111	0.000	0.000
5	0.000	0.200	0.000	5.000	5	0.000	0.000	0.111
6	0.000	0.000	0.000	9.000	0.000	6	3.000	0.000
7	0.000	0.333	0.000	0.000	0.000	0.333	5	0.111
8	9.000	0.000	0.000	0.000	9.000	0.000	9.000	5
column sums	16.000	6.533	8.000	20.000	19.200	6.444	20.000	5.333

Converge Matrix

Source: Product work of the author.

Figure 72 Calculations part 'b' for test 9.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0428
2	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.1744
3	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.1698
4	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0107
5	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0437
6	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.1164
7	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0465
8	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.3957
									##############	

stop delta	0.000005	eigenvalue	8.027006947
delta	0.000005	C.I.	0.003858
power	77	C.R.	0.002756

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.245	0.252	4.004	0.979	0.368	0.920	0.111
2	4.074	1.000	1.000	16.310	5.000	1.499	3.000	0.441
3	3.965	1.000	1.000	15.875	3.884	1.459	3.647	0.429
4	0.250	0.061	0.063	1.000	0.200	0.111	0.230	0.027
5	1.021	0.200	0.257	5.000	1.000	0.376	0.939	0.111
6	2.718	0.667	0.686	9.000	2.662	1.000	3.000	0.294
7	1.087	0.333	0.274	4.353	1.065	0.333	1.000	0.111
8	9.000	2.269	2.331	37.002	9.000	3.400	9.000	1.000
	23.115	5.776	5.863	92.544	23.790	8.545	21.736	2.524

		stop delta	0.000005	eigenvalue	8.026964772
RI	0.22947	delta	0.000000	C.I.	0.00385211
		power	6	C.R.	0.016786771

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	33215.261	8256.078	8376.134	134268.845	32907.834	12324.758	30874.343	3595.326	263818.579	0.0428
2	135312.588	33633.673	34122.762	546985.499	134060.190	50208.697	125776.137	14646.667	1074746.214	0.1744
3	131709.057	32737.972	33214.033	532418.614	130490.013	48871.580	122426.574	14256.610	1046124.452	0.1698
4	8295.883	2062.048	2092.033	33535.144	8219.099	3078.246	7711.213	897.973	65891.638	0.0107
5	33912.319	8429.341	8551.917	137086.624	33598.439	12583.407	31522.276	3670.778	269355.102	0.0437
6	90285.482	22441.613	22767.948	364968.600	89449.839	33501.067	83922.416	9772.789	717109.753	0.1164
7	36110.967	8975.843	9106.366	145974.407	35776.741	13399.230	33565.966	3908.767	286818.286	0.0465
8	306964.098	76299.855	77409.374	#######	304122.966	113901.206	285330.136	33226.776	2438121.231	0.3957
									6161985.255	

Source: Product work of the author.

Figure 73 Schedule for test 10.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0					1		1
2		0	1			1		
3		1	0			1		
4				0	1			
5				1	0		1	
6	1	1	1			0		1
7					1		0	1
8	1					1	1	0

generate schedule

Unlock

vertex	degree
1	2
2	2
3	1
4	1
5	2
6	2
7	2
8	2

Comparison

1	4	5
2	7	8
3	1	6
4	5	7
5	1	8
6	2	6
7	2	3

Random Redundant Comparisons

8	3	6
9	6	8

#	row	column	length
1	1	6	1
2	1	8	1
3	2	3	1
4	2	6	1
5	4	5	1
6	5	7	1
7	7	8	1
8	1	2	2
9	1	7	2
10	3	6	2
11	4	7	2
12	5	8	2
13	6	8	2
14	1	3	3
15	1	5	3
16	2	8	3
17	4	8	3
18	6	7	3
19	1	4	4
20	2	7	4
21	3	8	4
22	5	6	4
23	2	5	5
24	3	7	5
25	4	6	5
26	2	4	6
27	3	5	6
28	3	4	7

Source: Product work of the author.

Figure 74 Calculations part 'a' for test 10.

Test 10 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	1	0	1
2	0	0	1	0	0	1	0	0
3	0	1	0	0	0	1	0	0
4	0	0	0	0	1	0	0	0
5	0	0	0	1	0	0	1	0
6	1	1	1	0	0	0	0	1
7	0	0	0	0	1	0	0	1
8	1	0	0	0	0	1	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.000	0.000	0.111	0.000	0.111	5
2	0.000	1	1.000	0.000	0.000	0.200	0.000	0.000	5
3	0.000	1.000	1	0.000	0.000	0.200	0.000	0.000	5
4	0.000	0.000	0.000	1	0.143	0.000	0.000	0.000	6
5	0.000	0.000	0.000	7.000	1	0.000	7.000	0.000	5
6	9.000	5.000	5.000	0.000	0.000	1	0.000	0.200	3
7	0.000	0.000	0.000	0.000	0.143	0.000	1	0.143	5
8	9.000	0.000	0.000	0.000	0.000	5.000	7.000	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	6	0.000	0.000	0.000	0.000	0.111	0.000	0.111
2	0.000	6	1.000	0.000	0.000	0.200	0.000	0
3	0.000	1.000	6	0.000	0.000	0.200	0.000	0.000
4	0.000	0.000	0.000	7	0.143	0.000	0.000	0.000
5	0.000	0.000	0.000	7.000	6	0.000	7.000	0.000
6	9.000	5.000	5.000	0.000	0.000	4	0.000	0.200
7	0.000	0.000	0.000	0.000	0.143	0.000	6	0.143
8	9.000	0.000	0.000	0.000	0.000	5.000	7.000	5
column sums	24.000	12.000	12.000	14.000	6.286	9.511	20.000	5.454

Converge Matrix

Source: Product work of the author.

Figure 75 Calcualtions part 'b' for test 10.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0303
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0266
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0266
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0323
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.2564
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1512
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0460
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.4307
									###############	

stop delta	0.000005	eigenvalue	8.135135845
delta	0.000005	C.I.	0.019305
power	122	C.R.	0.013789

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	1.137	1.137	0.939	0.118	0.111	0.659	0.111
2	0.880	1.000	1.000	0.826	0.104	0.200	0.580	0.062
3	0.880	1.000	1.000	0.826	0.104	0.200	0.580	0.062
4	1.065	1.211	1.211	1.000	0.143	0.213	0.702	0.075
5	8.468	9.625	9.625	7.000	1.000	1.696	7.000	0.595
6	9.000	5.000	5.000	4.687	0.590	1.000	3.288	0.200
7	1.518	1.726	1.726	1.425	0.143	0.304	1.000	0.143
8	9.000	16.166	16.166	13.353	1.680	5.000	7.000	1.000
	31.811	36.864	36.864	30.056	3.881	8.725	20.808	2.248

		stop delta	0.000005	eigenvalue	8.135124098
RI	0.22947	delta	0.000000	C.I.	0.019303443
		power	8	C.R.	0.084120767

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	#######	#######	#######	#######	276103.722	506874.694	#######	175779.227	12461882.270	0.0303
2	#######	#######	#######	#######	242906.394	445930.766	#######	154644.395	10963527.668	0.0266
3	#######	#######	#######	#######	242906.394	445930.766	#######	154644.395	10963527.668	0.0266
4	#######	#######	#######	#######	294115.281	539940.700	#######	187246.117	13274829.799	0.0323
5	#######	#######	#######	#######	#######	#######	#######	#######	105530243.666	0.2564
6	#######	#######	#######	#######	#######	#######	#######	877515.415	62211552.635	0.1512
7	#######	#######	#######	3311896.167	419164.928	769508.403	#######	266857.990	18918918.749	0.0460
8	#######	#######	#######	#######	#######	#######	#######	#######	177239796.840	0.4306
									411564279.296	

Source: Product work of the author.

Figure 76 Schedule for test 11.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0			1	1		1	
2		0					1	1
3			0	1				1
4	1		1	0				
5	1				0	1		
6					1	0		
7	1	1					0	1
8		1	1				1	0

generate s chedule

Unlock

vertex	degree
1	2
2	2
3	2
4	1
5	2
6	1
7	2
8	2

Comparison

1	2	8
2	3	4
3	1	7
4	3	8
5	2	7
6	1	5
7	5	6

Random Redundant Comparisons

8	7	8
9	1	4

#	row	column	length
1	1	5	1
2	1	7	1
3	2	7	1
4	2	8	1
5	3	4	1
6	3	8	1
7	5	6	1
8	1	2	2
9	1	6	2
10	2	3	2
11	4	8	2
12	5	7	2
13	7	8	2
14	1	8	3
15	2	4	3
16	2	5	3
17	3	7	3
18	6	7	3
19	1	3	4
20	2	6	4
21	4	7	4
22	5	8	4
23	1	4	5
24	3	5	5
25	6	8	5
26	3	6	6
27	4	5	6
28	4	6	7

Source: Product work of the author.

Figure 77 Calculations part 'a' for test 11.

Test 11 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	1	1	0	1	0
2	0	0	0	0	0	0	1	1
3	0	0	0	1	0	0	0	1
4	1	0	1	0	0	0	0	0
5	1	0	0	0	0	1	0	0
6	0	0	0	0	1	0	0	0
7	1	1	0	0	0	0	0	1
8	0	1	1	0	0	0	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.200	0.200	0.000	0.200	0.000	4
2	0.000	1	0.000	0.000	0.000	0.000	1.000	0.200	5
3	0.000	0.000	1	0.333	0.000	0.000	0.000	0.143	5
4	5.000	0.000	3.000	1	0.000	0.000	0.000	0.000	5
5	5.000	0.000	0.000	0.000	1	0.143	0.000	0.000	5
6	0.000	0.000	0.000	0.000	7.000	1	0.000	0.000	6
7	5.000	1.000	0.000	0.000	0.000	0.000	1	0.143	4
8	0.000	5.000	7.000	0.000	0.000	0.000	7.000	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	5	0.000	0.000	0.200	0.200	0.000	0.200	0.000
2	0.000	6	0.000	0.000	0.000	0.000	1.000	0.2
3	0.000	0.000	6	0.333	0.000	0.000	0.000	0.143
4	5.000	0.000	3.000	6	0.000	0.000	0.000	0.000
5	5.000	0.000	0.000	0.000	6	0.143	0.000	0.000
6	0.000	0.000	0.000	0.000	7.000	7	0.000	0.000
7	5.000	1.000	0.000	0.000	0.000	0.000	5	0.143
8	0.000	5.000	7.000	0.000	0.000	0.000	7.000	5
column sums	20.000	12.000	16.000	6.533	13.200	7.143	13.200	5.486

Converge Matrix

Source: Product work of the author.

Figure 78 Calculations part 'b' for test 11.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0129
2	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0548
3	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0339
4	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0818
5	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0607
6	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.4117
7	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0531
8	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.2912
									##############	

stop delta	0.000005	eigenvalue	8.031782802
delta	0.000005	C.I.	0.004540
power	113	C.R.	0.003243

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.235	0.381	0.200	0.200	0.031	0.200	0.044
2	4.247	1.000	1.616	0.670	0.903	0.133	1.000	0.200
3	2.627	0.619	1.000	0.333	0.559	0.082	0.639	0.143
4	5.000	1.493	3.000	1.000	1.348	0.199	1.541	0.281
5	5.000	1.108	1.790	0.742	1.000	0.143	1.144	0.208
6	31.908	7.514	12.144	5.032	7.000	1.000	7.757	1.414
7	5.000	1.000	1.566	0.649	0.874	0.129	1.000	0.143
8	22.569	5.000	7.000	3.559	4.798	0.707	7.000	1.000
	77.350	17.969	28.497	12.185	16.681	2.425	20.281	3.433

	stop delta	0.000005	eigenvalue	8.031697021
RI 0.22947	delta	0.000000	C.I.	0.004528146
	power	6	C.R.	0.019732807

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	33699.553	7843.849	12813.450	5319.138	7080.324	1043.440	8194.165	1494.447	77488.366	0.0129
2	143105.709	33309.030	54412.517	22587.804	30066.712	4430.986	34796.656	6346.191	329055.606	0.0548
3	88540.330	20608.488	33665.336	13975.204	18602.448	2741.477	21528.893	3926.425	203588.600	0.0339
4	213681.597	49736.149	81247.310	33727.500	44894.807	6616.230	51957.435	9475.963	491336.991	0.0818
5	158530.482	36899.273	60277.418	25022.450	33307.478	4908.584	38547.243	7030.221	364523.148	0.0607
6	#######	250288.305	408862.605	169727.645	225925.111	33294.997	261466.516	47686.091	2472565.845	0.4117
7	138622.959	32265.634	52708.062	21880.249	29124.879	4292.187	33706.657	6147.398	318748.024	0.0531
8	760547.967	177023.779	289180.115	120044.883	159792.191	23548.868	184929.896	33727.391	1748795.090	0.2912
									6006101.670	

Source: Product work of the author.

Figure 79 Schedule for test 12.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0	1	1			1		
2	1	0					1	
3	1		0					1
4				0				1
5					0	1	1	
6	1				1	0	1	
7		1			1	1	0	
8			1	1				0

generate schedule

Unlock

Comparison

1	5	7
2	5	6
3	4	8
4	3	8
5	2	7
6	1	3
7	1	6

Random Redundant Comparisons

8	6	7
9	1	2

vertex	degree
1	2
2	1
3	2
4	1
5	2
6	2
7	2
8	2

#	row	column	length
1	1	3	1
2	1	6	1
3	2	7	1
4	3	8	1
5	4	8	1
6	5	6	1
7	5	7	1
8	1	5	2
9	1	8	2
10	2	5	2
11	3	4	2
12	3	6	2
13	6	7	2
14	1	4	3
15	1	7	3
16	2	6	3
17	3	5	3
18	6	8	3
19	1	2	4
20	3	7	4
21	4	6	4
22	5	8	4
23	2	3	5
24	4	5	5
25	7	8	5
26	2	8	6
27	4	7	6
28	2	4	7

Source: Product work of the author.

Figure 80 Calculations part 'a' for test 12.

Test 12 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	1	1	0	0	1	0	0
2	1	0	0	0	0	0	1	0
3	1	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0	1
5	0	0	0	0	0	1	1	0
6	1	0	0	0	1	0	1	0
7	0	1	0	0	1	1	0	0
8	0	0	1	1	0	0	0	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.33	0.333	0.000	0.000	0.143	0.000	0.000	4
2	3.000	1	0.000	0.000	0.000	0.000	0.333	0.000	5
3	3.000	0.000	1	0.000	0.000	0.000	0.000	0.143	5
4	0.000	0.000	0.000	1	0.000	0.000	0.000	0.200	6
5	0.000	0.000	0.000	0.000	1	0.200	0.333	0.000	5
6	7.000	0.000	0.000	0.000	5.000	1	5.000	0.000	4
7	0.000	3.000	0.000	0.000	3.000	0.200	1	0.000	4
8	0.000	0.000	7.000	5.000	0.000	0.000	0.000	1	5

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	5	0.333	0.333	0.000	0.000	0.143	0.000	0.000
2	3.000	6	0.000	0.000	0.000	0.000	0.333	0
3	3.000	0.000	6	0.000	0.000	0.000	0.000	0.143
4	0.000	0.000	0.000	7	0.000	0.000	0.000	0.200
5	0.000	0.000	0.000	0.000	6	0.200	0.333	0.000
6	7.000	0.000	0.000	0.000	5.000	5	5.000	0.000
7	0.000	3.000	0.000	0.000	3.000	0.200	5	0.000
8	0.000	0.000	7.000	5.000	0.000	0.000	0.000	6
column sums	18.000	9.333	13.333	12.000	14.000	5.543	10.667	6.343

Converge Matrix

Source: Product work of the author.

Figure 81 Calculations part 'b' for test 12.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0267
2	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0556
3	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0578
4	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0539
5	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0487
6	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.3256
7	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.1196
8	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.3121
									##############	

stop delta	0.000005	eigenvalue	8.158271307
delta	0.000005	C.I.	0.022610
power	89	C.R.	0.016150

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.333	0.333	0.495	0.549	0.143	0.223	0.086
2	3.000	1.000	0.962	1.032	1.142	0.171	0.333	0.178
3	3.000	1.039	1.000	1.072	1.187	0.177	0.483	0.143
4	2.018	0.969	0.933	1.000	1.107	0.165	0.450	0.200
5	1.823	0.875	0.842	0.903	1.000	0.200	0.333	0.156
6	7.000	5.858	5.637	6.044	5.000	1.000	5.000	1.043
7	4.482	3.000	2.071	2.221	3.000	0.200	1.000	0.383
8	11.693	5.615	7.000	5.000	6.415	0.959	2.609	1.000
	34.017	18.691	18.779	17.767	19.400	3.015	10.432	3.189

RI 0.22947

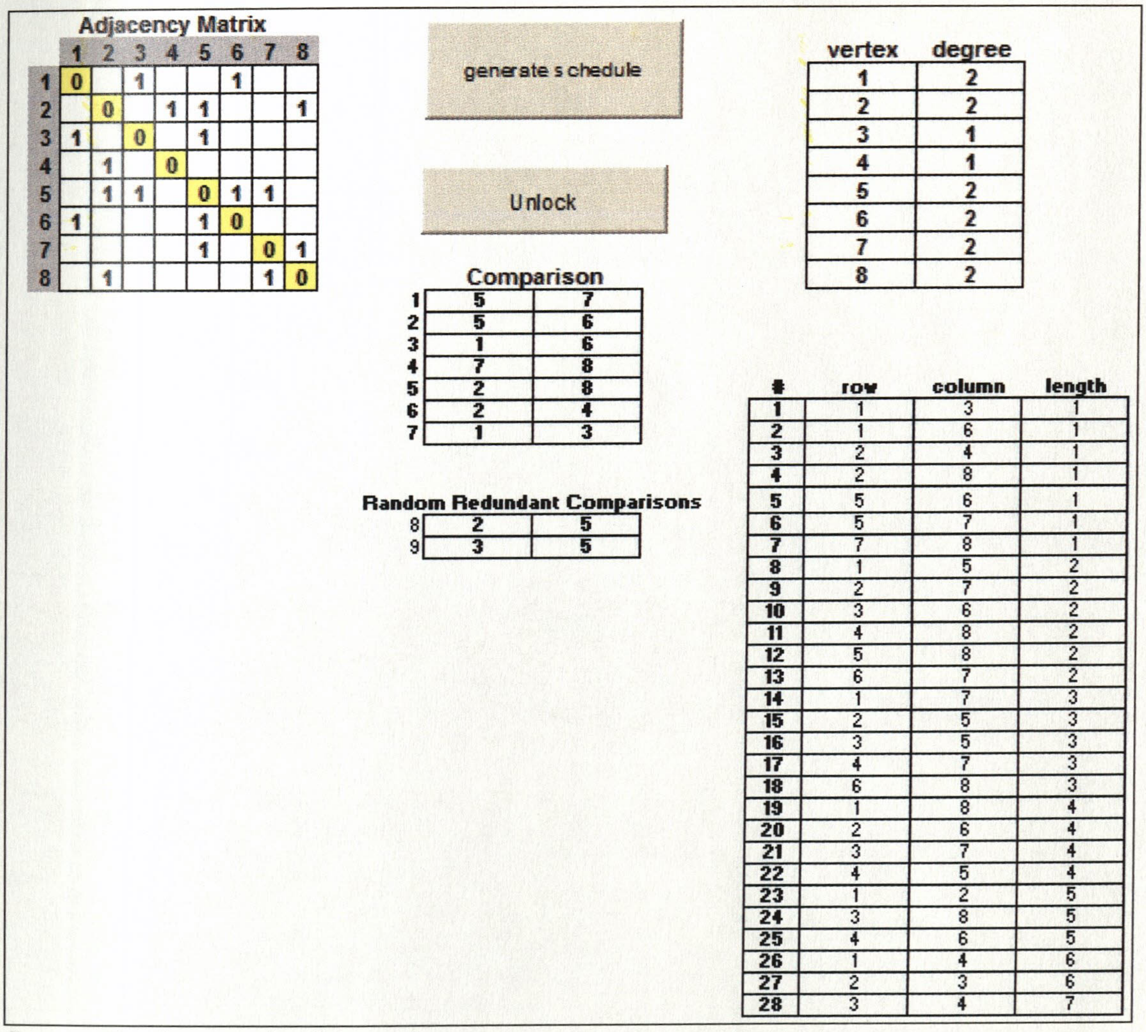
stop delta	0.000005	eigenvalue	8.158252312
delta	0.000001	C.I.	0.022607473
power	8	C.R.	0.09851911

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	#######	#######	1111729.820	1171849.609	#######	212482.162	565368.953	203883.752	8262090.058	0.0267
2	#######	#######	#######	#######	#######	442470.882	1177319.868	424565.762	17204903.812	0.0556
3	#######	#######	#######	#######	#######	459810.608	#######	441203.777	17879134.682	0.0578
4	#######	#######	#######	#######	#######	428910.349	1141237.929	411553.868	16677615.318	0.0539
5	#######	#######	#######	#######	#######	387319.791	#######	371646.345	15060422.264	0.0487
6	#######	#######	#######	#######	#######	#######	#######	#######	100788238.598	0.3256
7	#######	#######	#######	#######	#######	952398.393	#######	913858.280	37032760.614	0.1196
8	#######	#######	#######	#######	#######	#######	6611317.425	#######	96615263.169	0.3121
									309520428.514	

Source: Product work of the author.

Figure 82 Schedule for test 13.



Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0		1			1		
2		0		1	1			1
3	1		0		1			
4		1		0				
5		1	1		0	1	1	
6	1				1	0		
7					1		0	1
8		1					1	0

vertex	degree
1	2
2	2
3	1
4	1
5	2
6	2
7	2
8	2

Comparison

1	5	7
2	5	6
3	1	6
4	7	8
5	2	8
6	2	4
7	1	3

Random Redundant Comparisons

8	2	5
9	3	5

#	row	column	length
1	1	3	1
2	1	6	1
3	2	4	1
4	2	8	1
5	5	6	1
6	5	7	1
7	7	8	1
8	1	5	2
9	2	7	2
10	3	6	2
11	4	8	2
12	5	8	2
13	6	7	2
14	1	7	3
15	2	5	3
16	3	5	3
17	4	7	3
18	6	8	3
19	1	8	4
20	2	6	4
21	3	7	4
22	4	5	4
23	1	2	5
24	3	8	5
25	4	6	5
26	1	4	6
27	2	3	6
28	3	4	7

Source: Product work of the author.

Figure 83 Calculations part 'a' for test 13.

Test 13 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	1	0	0	1	0	0
2	0	0	0	1	1	0	0	1
3	1	0	0	0	1	0	0	0
4	0	1	0	0	0	0	0	0
5	0	1	1	0	0	1	1	0
6	1	0	0	0	1	0	0	0
7	0	0	0	0	1	0	0	1
8	0	1	0	0	0	0	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.333	0.000	0.000	0.200	0.000	0.000	5
2	0.000	1	0.000	3.000	0.200	0.000	0.000	0.143	4
3	3.000	0.000	1	0.000	0.200	0.000	0.000	0.000	5
4	0.000	0.333	0.000	1	0.000	0.000	0.000	0.000	6
5	0.000	5.000	5.000	0.000	1	0.143	5.000	0.000	3
6	5.000	0.000	0.000	0.000	7.000	1	0.000	0.000	5
7	0.000	0.000	0.000	0.000	0.200	0.000	1	0.143	5
8	0.000	7.000	0.000	0.000	0.000	0.000	7.000	1	5

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	6	0.000	0.333	0.000	0.000	0.200	0.000	0.000
2	0.000	5	0.000	3.000	0.200	0.000	0.000	0.14286
3	3.000	0.000	6	0.000	0.200	0.000	0.000	0.000
4	0.000	0.333	0.000	7	0.000	0.000	0.000	0.000
5	0.000	5.000	5.000	0.000	4	0.143	5.000	0.000
6	5.000	0.000	0.000	0.000	7.000	6	0.000	0.000
7	0.000	0.000	0.000	0.000	0.200	0.000	6	0.143
8	0.000	7.000	0.000	0.000	0.000	0.000	7.000	6
column sums	14.000	17.333	11.333	10.000	11.600	6.343	18.000	6.286

Converge Matrix

Source: Product work of the author.

Figure 84 Calculations part 'b' for test 13.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0580
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0166
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0844
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0039
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1535
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.5628
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0187
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1021
									###############	

stop delta	0.000005	eigenvalue	8.424616475
delta	0.000005	C.I.	0.060659
power	73	C.R.	0.043328

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	3.485	0.333	14.860	0.378	0.200	3.106	0.568
2	0.287	1.000	0.197	3.000	0.200	0.030	0.891	0.143
3	3.000	5.071	1.000	21.625	0.200	0.150	4.520	0.827
4	0.067	0.333	0.046	1.000	0.025	0.007	0.209	0.038
5	2.646	5.000	5.000	39.315	1.000	0.143	5.000	1.504
6	5.000	33.802	6.665	144.136	7.000	1.000	30.126	5.514
7	0.322	1.122	0.221	4.784	0.200	0.033	1.000	0.143
8	1.759	7.000	1.209	26.140	0.665	0.181	7.000	1.000
	14.081	56.814	14.672	254.860	9.668	1.744	51.852	9.738

RI 0.22947

stop delta	0.000005	eigenvalue	8.424001204
delta	0.000002	C.I.	0.060571601
power	9	C.R.	0.263959627

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	#######	#######	#######	#######	#######	#######	#######	#######	605170290.019	0.0580
2	#######	#######	#######	#######	#######	790442.440	#######	#######	173624128.606	0.0166
3	#######	#######	#######	#######	#######	#######	#######	#######	880686068.312	0.0844
4	#######	#######	#######	#######	773211.890	185323.554	#######	930876.448	40707115.129	0.0039
5	#######	#######	#######	#######	#######	#######	#######	#######	1601085722.279	0.1535
6	#######	#######	#######	#######	#######	#######	#######	#######	5869815749.864	0.5628
7	#######	#######	#######	#######	#######	886955.752	#######	#######	194823705.543	0.0187
8	#######	#######	#######	#######	#######	#######	#######	#######	1064303740.089	0.1020
									###############	

Source: Product work of the author.

Figure 85 Schedule for test 14.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0	1	1					1
2	1	0						
3	1		0	1			1	
4			1	0		1		
5					0	1	1	
6				1	1	0		1
7			1		1		0	
8	1					1		0

generate s chedule

Unlock

Comparison

1	5	7
2	5	6
3	6	8
4	3	4
5	1	8
6	3	7
7	1	2

Random Redundant Comparisons

8	4	6
9	1	3

vertex	degree
1	2
2	1
3	2
4	1
5	2
6	2
7	2
8	2

#	row	column	length
1	1	2	1
2	1	8	1
3	3	4	1
4	3	7	1
5	5	6	1
6	5	7	1
7	6	8	1
8	1	6	2
9	2	8	2
10	3	5	2
11	4	7	2
12	5	8	2
13	6	7	2
14	1	5	3
15	2	6	3
16	3	6	3
17	4	5	3
18	7	8	3
19	1	7	4
20	2	5	4
21	3	8	4
22	4	6	4
23	1	3	5
24	2	7	5
25	4	8	5
26	1	4	6
27	2	3	6
28	2	4	7

Source: Product work of the author.

Figure 86 Calculations part 'a' for test 14.

Test 14 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	0	1
2	1	0	0	0	0	0	0	0
3	1	0	0	1	0	0	1	0
4	0	0	1	0	0	1	0	0
5	0	0	0	0	0	1	1	0
6	0	0	0	1	1	0	0	1
7	0	0	1	0	1	0	0	0
8	1	0	0	0	0	1	0	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.33	0.333	0.000	0.000	0.000	0.000	0.143	4
2	3.000	1	0.000	0.000	0.000	0.000	0.000	0.000	6
3	3.000	0.000	1	0.333	0.000	0.000	0.200	0.000	4
4	0.000	0.000	3.000	1	0.000	0.200	0.000	0.000	5
5	0.000	0.000	0.000	0.000	1	0.143	0.333	0.000	5
6	0.000	0.000	0.000	5.000	7.000	1	0.000	5.000	4
7	0.000	0.000	5.000	0.000	3.000	0.000	1	0.000	5
8	7.000	0.000	0.000	0.000	0.000	0.200	0.000	1	5

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	5	0.333	0.333	0.000	0.000	0.000	0.000	0.143
2	3.000	7	0.000	0.000	0.000	0.000	0.000	0
3	3.000	0.000	5	0.333	0.000	0.000	0.200	0.000
4	0.000	0.000	3.000	6	0.000	0.200	0.000	0.000
5	0.000	0.000	0.000	0.000	6	0.143	0.333	0.000
6	0.000	0.000	0.000	5.000	7.000	5	0.000	5.000
7	0.000	0.000	5.000	0.000	3.000	0.000	6	0.000
8	7.000	0.000	0.000	0.000	0.000	0.200	0.000	6
column sums	18.000	7.333	13.333	11.333	16.000	5.543	6.533	11.143

Converge Matrix

Source: Product work of the author.

Figure 87 Calculations part 'b' for test 14.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0125
2	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0373
3	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0362
4	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.1012
5	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0647
6	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.4702
7	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.1873
8	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0906
									##############	

stop delta	0.000005	eigenvalue	8.002319288
delta	0.000005	C.I.	0.000331
power	102	C.R.	0.000237

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.333	0.333	0.123	0.193	0.027	0.067	0.143
2	3.000	1.000	1.032	0.369	0.577	0.079	0.199	0.412
3	3.000	0.969	1.000	0.333	0.559	0.077	0.200	0.399
4	8.111	2.709	3.000	1.000	1.563	0.200	0.540	1.117
5	5.189	1.733	1.789	0.640	1.000	0.143	0.333	0.715
6	37.700	12.592	13.000	5.000	7.000	1.000	2.511	5.000
7	15.016	5.015	5.000	1.851	3.000	0.398	1.000	2.068
8	7.000	2.425	2.504	0.895	1.399	0.200	0.484	1.000
	80.017	26.776	27.659	10.212	15.291	2.124	5.334	10.854

RI 0.22947	stop delta	0.000005	eigenvalue	8.002258719
	delta	0.000004	C.I.	0.000322674
	power	4	C.R.	0.001406153

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	512.451	171.103	176.806	63.242	98.748	13.602	34.125	70.568	1140.645	0.0125
2	1534.295	512.289	529.365	189.348	295.655	40.724	102.172	211.283	3415.131	0.0373
3	1486.171	496.221	512.761	183.409	286.382	39.447	98.967	204.656	3308.013	0.0362
4	4156.424	1387.799	1434.056	512.946	800.934	110.323	276.785	572.368	9251.634	0.1012
5	2659.394	887.952	917.548	328.197	512.459	70.588	177.094	366.217	5919.449	0.0647
6	19319.988	6450.798	6665.809	2384.288	3722.918	512.805	1286.556	2660.495	43003.658	0.4702
7	7695.404	2569.437	2655.079	949.693	1482.887	204.257	512.452	1059.710	17128.920	0.1873
8	3721.417	1242.553	1283.969	459.262	717.109	98.776	247.817	512.465	8283.366	0.0906
									91450.816	

Source: Product work of the author.

Figure 88 Schedule for test 15.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0			1			1	
2		0	1		1			
3		1	0	1				
4	1		1	0		1	1	
5		1			0		1	
6				1		0		1
7	1			1	1		0	
8						1		0

generate schedule

Unlock

vertex	degree
1	1
2	2
3	2
4	2
5	2
6	2
7	2
8	1

Comparison

1	3	4
2	2	3
3	5	7
4	2	5
5	1	7
6	6	8
7	4	6

Random Redundant Comparisons

8	4	7
9	1	4

#	row	column	length
1	1	7	1
2	2	3	1
3	2	5	1
4	3	4	1
5	4	6	1
6	5	7	1
7	6	8	1
8	1	5	2
9	2	4	2
10	2	7	2
11	3	5	2
12	3	6	2
13	4	8	2
14	1	2	3
15	2	6	3
16	3	7	3
17	3	8	3
18	4	5	3
19	1	3	4
20	2	8	4
21	4	7	4
22	5	6	4
23	1	4	5
24	5	8	5
25	6	7	5
26	1	6	6
27	7	8	6
28	1	8	7

Source: Product work of the author.

Figure 89 Calculations part 'a' for test 15.

Test 15 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	1	0	0	1	0
2	0	0	1	0	1	0	0	0
3	0	1	0	1	0	0	0	0
4	1	0	1	0	0	1	1	0
5	0	1	0	0	0	0	1	0
6	0	0	0	1	0	0	0	1
7	1	0	0	1	1	0	0	0
8	0	0	0	0	0	1	0	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.200	0.000	0.000	0.143	0.000	5
2	0.000	1	0.200	0.000	0.143	0.000	0.000	0.000	5
3	0.000	5.000	1	9.000	0.000	0.000	0.000	0.000	5
4	5.000	0.000	0.111	1	0.000	0.143	0.143	0.000	3
5	0.000	7.000	0.000	0.000	1	0.000	5.000	0.000	5
6	0.000	0.000	0.000	7.000	0.000	1	0.000	0.200	5
7	7.000	0.000	0.000	7.000	0.200	0.000	1	0.000	4
8	0.000	0.000	0.000	0.000	0.000	5.000	0.000	1	6

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	6	0.000	0.000	0.200	0.000	0.000	0.143	0.000
2	0.000	6	0.200	0.000	0.143	0.000	0.000	0
3	0.000	5.000	6	9.000	0.000	0.000	0.000	0.000
4	5.000	0.000	0.111	4	0.000	0.143	0.143	0.000
5	0.000	7.000	0.000	0.000	6	0.000	5.000	0.000
6	0.000	0.000	0.000	7.000	0.000	6	0.000	0.200
7	7.000	0.000	0.000	7.000	0.200	0.000	5	0.000
8	0.000	0.000	0.000	0.000	0.000	5.000	0.000	7
column sums	18.000	18.000	6.311	27.200	6.343	11.143	10.286	7.200

Converge Matrix

Source: Product work of the author.

Figure 90 Calculations part 'b' for test 15.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0059
2	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0298
3	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.1386
4	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0163
5	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.2517
6	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0910
7	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.0657
8	#########	#########	#########	#########	#########	#########	#########	#########	##############	0.4010
									##############	

stop delta	0.000005	eigenvalue	8.134778496
delta	0.000005	C.I.	0.019254
power	107	C.R.	0.013753

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	0.199	0.043	0.200	0.024	0.065	0.143	0.015
2	5.036	1.000	0.200	1.830	0.143	0.328	0.454	0.074
3	23.399	5.000	1.000	9.000	0.551	1.523	2.110	0.346
4	5.000	0.547	0.111	1.000	0.065	0.143	0.143	0.041
5	42.489	7.000	1.816	15.438	1.000	2.765	5.000	0.628
6	15.367	3.051	0.657	7.000	0.362	1.000	1.386	0.200
7	7.000	2.202	0.474	7.000	0.200	0.722	1.000	0.164
8	67.699	13.444	2.893	24.599	1.593	5.000	6.105	1.000
	166.989	32.442	7.194	66.067	3.937	11.545	16.340	2.467

RI	0.22947	stop delta	0.000005	eigenvalue	8.134718509
		delta	0.000004	C.I.	0.019245501
		power	7	C.R.	0.08386827

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	307162.254	56873.102	12187.032	113204.748	6782.843	18661.037	27627.604	4218.374	546716.993	0.0059
2	#######	286400.025	61371.138	570073.323	34156.853	93972.800	139126.614	21242.775	2753139.893	0.0298
3	7187179.194	#######	285160.599	#######	158709.606	436644.017	646451.069	98704.419	12792446.012	0.1386
4	845355.616	156523.807	33540.660	311558.050	18667.449	51358.192	76035.740	11609.636	1504649.149	0.0163
5	#######	#######	517813.872	#######	288195.666	792887.577	#######	179234.158	23229398.875	0.2517
6	#######	873958.557	187275.940	#######	104230.710	286760.946	424549.485	64822.990	8401290.183	0.0910
7	#######	630709.294	135151.357	#######	75220.161	206946.677	306384.890	46780.784	6062954.260	0.0657
8	#######	#######	825073.303	#######	459204.607	#######	#######	285587.777	37013201.739	0.4010
									92303797.104	

Source: Product work of the author.

Figure 91 Schedule for test 16.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0						1	
2		0		1		1		1
3			0	1		1		
4		1	1	0				1
5					0	1		
6		1	1		1	0		
7	1						0	1
8		1		1			1	0

generate schedule

Unlock

vertex	degree
1	1
2	2
3	2
4	2
5	1
6	2
7	2
8	2

Comparison

1	2	8
2	2	4
3	3	4
4	1	7
5	7	8
6	3	6
7	5	6

Random Redundant Comparisons

8	4	8
9	2	6

#	row	column	length
1	1	7	1
2	2	4	1
3	2	8	1
4	3	4	1
5	3	6	1
6	5	6	1
7	7	8	1
8	1	8	2
9	2	3	2
10	2	7	2
11	3	5	2
12	4	6	2
13	4	8	2
14	1	2	3
15	2	6	3
16	3	8	3
17	4	5	3
18	4	7	3
19	1	4	4
20	2	5	4
21	3	7	4
22	6	8	4
23	1	3	5
24	5	8	5
25	6	7	5
26	1	6	6
27	5	7	6
28	1	5	7

Source: Product work of the author.

Figure 92 Calculation part 'a' for test 16.

Test 16 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	1	0
2	0	0	0	1	0	1	0	1
3	0	0	0	1	0	1	0	0
4	0	1	1	0	0	0	0	1
5	0	0	0	0	0	1	0	0
6	0	1	1	0	1	0	0	0
7	1	0	0	0	0	0	0	1
8	0	1	0	1	0	0	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.000	0.000	0.000	3.000	0.000	6
2	0.000	1	0.000	0.333	0.000	0.200	0.000	0.333	4
3	0.000	0.000	1	0.200	0.000	0.143	0.000	0.000	5
4	0.000	3.000	5.000	1	0.000	0.000	0.000	0.333	4
5	0.000	0.000	0.000	0.000	1	3.000	0.000	0.000	6
6	0.000	5.000	7.000	0.000	0.333	1	0.000	0.000	4
7	0.333	0.000	0.000	0.000	0.000	0.000	1	0.143	5
8	0.000	3.000	0.000	3.000	0.000	0.000	7.000	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	7	0.000	0.000	0.000	0.000	0.000	3.000	0.000
2	0.000	5	0.000	0.333	0.000	0.200	0.000	0.33333
3	0.000	0.000	6	0.200	0.000	0.143	0.000	0.000
4	0.000	3.000	5.000	5	0.000	0.000	0.000	0.333
5	0.000	0.000	0.000	0.000	7	3.000	0.000	0.000
6	0.000	5.000	7.000	0.000	0.333	5	0.000	0.000
7	0.333	0.000	0.000	0.000	0.000	0.000	6	0.143
8	0.000	3.000	0.000	3.000	0.000	0.000	7.000	5
column sums	7.333	16.000	18.000	8.533	7.333	8.343	16.000	5.810

Converge Matrix

Source: Product work of the author.

Figure 93 Calculations part 'b' for test 16.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	########	########	########	########	########	########	########	########	##############	0.0629
2	########	########	########	########	########	########	########	########	##############	0.0389
3	########	########	########	########	########	########	########	########	##############	0.0193
4	########	########	########	########	########	########	########	########	##############	0.0887
5	########	########	########	########	########	########	########	########	##############	0.4368
6	########	########	########	########	########	########	########	########	##############	0.1551
7	########	########	########	########	########	########	########	########	##############	0.0223
8	########	########	########	########	########	########	########	########	##############	0.1759
									##############	

stop delta	0.000005	eigenvalue	8.065077742
delta	0.000005	C.I.	0.009297
power	120	C.R.	0.006641

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	1.618	3.257	0.709	0.144	0.406	3.000	0.358
2	0.618	1.000	2.013	0.333	0.089	0.200	1.742	0.333
3	0.307	0.497	1.000	0.200	0.044	0.143	0.865	0.110
4	1.410	3.000	5.000	1.000	0.203	0.572	3.973	0.333
5	6.940	11.227	22.606	4.922	1.000	3.000	19.557	2.483
6	2.464	5.000	7.000	1.748	0.333	1.000	6.944	0.881
7	0.333	0.574	1.156	0.252	0.051	0.144	1.000	0.143
8	2.795	3.000	9.106	3.000	0.403	1.134	7.000	1.000
	15.868	25.916	51.138	12.165	2.268	6.599	44.081	5.641

RI 0.22947

stop delta	0.000005	eigenvalue	8.064972581
delta	0.000000	C.I.	0.009281797
power	6	C.R.	0.040448324

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	33863.766	56759.920	110626.502	24813.347	4878.931	13849.717	95534.026	12657.154	352983.363	0.0629
2	20933.673	35087.403	68386.340	15338.978	3016.025	8561.526	59056.551	7824.311	218204.806	0.0389
3	10396.962	17426.613	33964.903	7618.273	1497.945	4252.184	29331.163	3886.040	108374.084	0.0193
4	47745.624	80027.681	155975.880	34985.095	6878.963	19527.158	134696.556	17845.759	497682.715	0.0887
5	235032.496	393943.979	767806.594	172217.775	33862.368	96124.378	663056.811	87847.356	2449891.757	0.4368
6	83452.281	139876.450	272622.756	61148.836	12023.409	34130.587	235429.588	31191.724	869875.632	0.1551
7	12017.087	20142.140	39257.544	8805.407	1731.365	4914.789	33901.740	4491.588	125261.660	0.0223
8	94669.729	158678.584	309268.109	69368.310	13639.565	38718.358	267075.473	35384.332	986802.461	0.1759
									5609076.477	

Source: Product work of the author.

Figure 94 Schedule for test 17.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0				1			1
2		0		1				
3			0			1		1
4		1		0			1	
5	1				0	1		1
6			1		1	0	1	
7				1		1	0	
8	1		1		1			0

generate schedule

Unlock

vertex	degree
1	2
2	1
3	1
4	2
5	2
6	2
7	2
8	2

Comparison

1	5	6
2	6	7
3	4	7
4	2	4
5	1	8
6	3	8
7	1	5

Random Redundant Comparisons

8	3	6
9	5	8

#	row	column	length
1	1	5	1
2	1	8	1
3	2	4	1
4	3	8	1
5	4	7	1
6	5	6	1
7	6	7	1
8	1	3	2
9	1	6	2
10	2	7	2
11	4	6	2
12	5	7	2
13	5	8	2
14	1	7	3
15	2	6	3
16	3	5	3
17	4	5	3
18	6	8	3
19	1	4	4
20	2	5	4
21	3	6	4
22	7	8	4
23	1	2	5
24	3	7	5
25	4	8	5
26	2	8	6
27	3	4	6
28	2	3	7

Source: Product work of the author.

Figure 95 Calculations part 'a' for test 17.

Test 17 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	0	1	0	0	1
2	0	0	0	1	0	0	0	0
3	0	0	0	0	0	1	0	1
4	0	1	0	0	0	0	1	0
5	1	0	0	0	0	1	0	1
6	0	0	1	0	1	0	1	0
7	0	0	0	1	0	1	0	0
8	1	0	1	0	1	0	0	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.000	0.200	0.000	0.000	0.200	5
2	0.000	1	0.000	0.200	0.000	0.000	0.000	0.000	6
3	0.000	0.000	1	0.000	0.000	0.125	0.000	0.200	5
4	0.000	5.000	0.000	1	0.000	0.000	0.333	0.000	5
5	5.000	0.000	0.000	0.000	1	0.143	0.000	3.000	4
6	0.000	0.000	8.000	0.000	7.000	1	9.000	0.000	4
7	0.000	0.000	0.000	3.000	0.000	0.111	1	0.000	5
8	5.000	0.000	5.000	0.000	0.333	0.000	0.000	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	6	0.000	0.000	0.000	0.200	0.000	0.000	0.200
2	0.000	7	0.000	0.200	0.000	0.000	0.000	0
3	0.000	0.000	6	0.000	0.000	0.125	0.000	0.200
4	0.000	5.000	0.000	6	0.000	0.000	0.333	0.000
5	5.000	0.000	0.000	0.000	5	0.143	0.000	3.000
6	0.000	0.000	8.000	0.000	7.000	5	9.000	0.000
7	0.000	0.000	0.000	3.000	0.000	0.111	6	0.000
8	5.000	0.000	5.000	0.000	0.333	0.000	0.000	5
column sums	16.000	12.000	19.000	9.200	12.533	5.379	15.333	8.400

Converge Matrix

Source: Product work of the author.

Figure 96 Calculations part 'b' for test 17.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0259
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0014
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0425
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0088
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1753
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.5848
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0399
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1215
									###############	

stop delta	0.000005	eigenvalue	8.293266139
delta	0.000005	C.I.	0.041895
power	113	C.R.	0.029925

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	18.956	0.609	2.946	0.200	0.044	0.650	0.200
2	0.053	1.000	0.032	0.200	0.008	0.002	0.034	0.011
3	1.641	31.102	1.000	4.834	0.242	0.125	1.066	0.200
4	0.339	5.000	0.207	1.000	0.050	0.015	0.333	0.072
5	5.000	128.406	4.129	19.958	1.000	0.143	4.400	3.000
6	22.592	428.248	8.000	66.563	7.000	1.000	9.000	4.812
7	1.540	29.185	0.938	3.000	0.227	0.111	1.000	0.328
8	5.000	88.989	5.000	13.832	0.333	0.208	3.049	1.000
	37.165	730.885	19.915	112.333	9.061	1.648	19.532	9.624

RI	0.22947	stop delta	0.000005	eigenvalue	8.291984468
		delta	0.000004	C.I.	0.041712067
		power	7	C.R.	0.181773331

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	316394.021	#######	206543.390	950616.957	53710.642	15783.711	213309.313	75119.237	7829945.224	0.0259
2	16676.914	316175.179	10886.714	50106.367	2831.067	831.950	11243.372	3959.478	412711.041	0.0014
3	519109.265	#######	338876.602	#######	88124.126	25896.144	349978.488	123251.036	12846679.706	0.0425
4	107329.847	#######	70064.874	322476.398	18220.365	5354.288	72360.418	25482.571	2656140.574	0.0088
5	#######	#######	1399111.521	#######	363836.135	106920.411	#######	508843.199	53039758.633	0.1753
6	#######	#######	#######	#######	1213353.139	356570.115	#######	#######	176886381.782	0.5848
7	487019.786	#######	317928.076	#######	82676.523	24295.423	328343.681	115631.321	12052529.454	0.0398
8	#######	#######	969596.122	#######	252149.180	74096.302	#######	352645.425	36757490.765	0.1215
									302481637.179	

Source: Product work of the author.

Figure 97 Schedule for test 18.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0					1		
2		0		1	1			1
3			0	1			1	
4		1	1	0				
5		1			0	1		
6	1				1	0		1
7			1				0	1
8		1				1	1	0

generate schedule

Unlock

vertex	degree
1	1
2	2
3	2
4	1
5	2
6	2
7	2
8	2

Comparison

1	3	4
2	5	6
3	2	5
4	7	8
5	3	7
6	2	8
7	1	6

Random Redundant Comparisons

8	6	8
9	2	4

#	row	column	length
1	1	6	1
2	2	5	1
3	2	8	1
4	3	4	1
5	3	7	1
6	5	6	1
7	7	8	1
8	1	5	2
9	2	6	2
10	2	7	2
11	3	8	2
12	4	7	2
13	5	8	2
14	1	2	3
15	2	3	3
16	4	8	3
17	5	7	3
18	6	8	3
19	1	8	4
20	2	4	4
21	3	5	4
22	6	7	4
23	1	7	5
24	3	6	5
25	4	5	5
26	1	3	6
27	4	6	6
28	1	4	7

Source: Product work of the author.

Figure 98 Calculations part 'a' for test 18.

Test 18 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	1	0	0
2	0	0	0	1	1	0	0	1
3	0	0	0	1	0	0	1	0
4	0	1	1	0	0	0	0	0
5	0	1	0	0	0	1	0	0
6	1	0	0	0	1	0	0	1
7	0	0	1	0	0	0	0	1
8	0	1	0	0	0	1	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.000	0.000	0.200	0.000	0.000	6
2	0.000	1	0.000	0.333	0.111	0.000	0.000	0.111	4
3	0.000	0.000	1	0.200	0.000	0.000	0.111	0.000	5
4	0.000	3.000	5.000	1	0.000	0.000	0.000	0.000	5
5	0.000	9.000	0.000	0.000	1	0.167	0.000	0.000	5
6	5.000	0.000	0.000	0.000	6.000	1	0.000	4.000	4
7	0.000	0.000	9.000	0.000	0.000	0.000	1	8.000	5
8	0.000	9.000	0.000	0.000	0.000	0.250	0.125	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	7	0.000	0.000	0.000	0.000	0.200	0.000	0.000
2	0.000	5	0.000	0.333	0.111	0.000	0.000	0.11111
3	0.000	0.000	6	0.200	0.000	0.000	0.111	0.000
4	0.000	3.000	5.000	6	0.000	0.000	0.000	0.000
5	0.000	9.000	0.000	0.000	6	0.167	0.000	0.000
6	5.000	0.000	0.000	0.000	6.000	5	0.000	4.000
7	0.000	0.000	9.000	0.000	0.000	0.000	6	8.000
8	0.000	9.000	0.000	0.000	0.000	0.250	0.125	5
column sums	12.000	26.000	20.000	6.533	12.111	5.617	6.236	17.111

Converge Matrix

Source: Product work of the author.

Figure 99 Calculations part 'b' for test 18.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0513
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0129
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0253
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0744
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0758
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.3136
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.3719
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0748
									###############	

stop delta	0.000005	eigenvalue	8.221249158
delta	0.000004	C.I.	0.031607
power	42	C.R.	0.022576

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	3.984	2.029	0.690	0.678	0.200	0.138	0.687
2	0.251	1.000	0.509	0.333	0.111	0.041	0.035	0.111
3	0.493	1.963	1.000	0.200	0.334	0.081	0.111	0.338
4	1.449	3.000	5.000	1.000	0.982	0.237	0.200	0.995
5	1.475	9.000	2.994	1.019	1.000	0.167	0.204	1.013
6	5.000	24.334	12.395	4.217	6.000	1.000	0.843	4.000
7	7.243	28.853	9.000	5.000	4.909	1.186	1.000	8.000
8	1.457	9.000	2.956	1.006	0.987	0.250	0.125	1.000
	18.367	81.134	35.883	13.465	15.001	3.161	2.656	16.144

RI 0.22947

stop delta	0.000005	eigenvalue	8.221230798
delta	0.000001	C.I.	0.0316044
power	8	C.R.	0.137726021

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	#######	#######	#######	1871840.861	#######	413231.604	362080.902	#######	24857083.304	0.0513
2	623221.573	#######	#######	469857.623	435818.713	103726.786	90887.258	448245.669	6239468.702	0.0129
3	#######	#######	#######	922375.086	855553.138	203625.471	178420.260	879948.157	12248664.151	0.0253
4	#######	#######	#######	2711171.034	#######	598523.727	524437.029	#######	36002946.102	0.0744
5	#######	#######	#######	#######	#######	609657.468	534192.857	#######	36672669.305	0.0758
6	#######	#######	#######	#######	#######	#######	#######	#######	151836621.882	0.3137
7	#######	#######	#######	#######	#######	#######	#######	#######	180028255.828	0.3719
8	#######	#######	#######	#######	#######	601844.680	527347.164	#######	36202707.419	0.0748
									484088416.692	

Source: Product work of the author.

Figure 100 Schedule for test 19.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0			1			1	1
2		0		1		1		1
3			0		1			
4	1	1		0				
5			1		0	1		
6		1			1	0		
7	1						0	1
8	1	1					1	0

generate schedule

Unlock

vertex	degree
1	2
2	2
3	1
4	1
5	2
6	2
7	2
8	2

Comparison

1	5	6
2	7	8
3	1	7
4	2	6
5	2	8
6	1	4
7	3	5

Random Redundant Comparisons

8	1	8
9	2	4

#	row	column	length
1	1	4	1
2	1	7	1
3	2	6	1
4	2	8	1
5	3	5	1
6	5	6	1
7	7	8	1
8	1	8	2
9	2	5	2
10	2	7	2
11	3	6	2
12	4	7	2
13	6	8	2
14	1	2	3
15	2	3	3
16	4	8	3
17	5	8	3
18	6	7	3
19	1	6	4
20	2	4	4
21	3	8	4
22	5	7	4
23	1	5	5
24	3	7	5
25	4	6	5
26	1	3	6
27	4	5	6
28	3	4	7

Source: Product work of the author.

Figure 101 Calculations part 'a' for test 19.

Test 19 light.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	1	0	0	1	1
2	0	0	0	1	0	1	0	1
3	0	0	0	0	1	0	0	0
4	1	1	0	0	0	0	0	0
5	0	0	1	0	0	1	0	0
6	0	1	0	0	1	0	0	0
7	1	0	0	0	0	0	0	1
8	1	1	0	0	0	0	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	3.000	0.000	0.000	0.200	0.200	4
2	0.000	1	0.000	1.000	0.000	0.200	0.000	0.200	4
3	0.000	0.000	1	0.000	0.200	0.000	0.000	0.000	6
4	0.333	1.000	0.000	1	0.000	0.000	0.000	0.000	5
5	0.000	0.000	5.000	0.000	1	5.000	0.000	0.000	5
6	0.000	5.000	0.000	0.000	0.200	1	0.000	0.000	5
7	5.000	0.000	0.000	0.000	0.000	0.000	1	0.333	5
8	5.000	5.000	0.000	0.000	0.000	0.000	3.000	1	4

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	5	0.000	0.000	3.000	0.000	0.000	0.200	0.200
2	0.000	5	0.000	1.000	0.000	0.200	0.000	0.2
3	0.000	0.000	7	0.000	0.200	0.000	0.000	0.000
4	0.333	1.000	0.000	6	0.000	0.000	0.000	0.000
5	0.000	0.000	5.000	0.000	6	5.000	0.000	0.000
6	0.000	5.000	0.000	0.000	0.200	6	0.000	0.000
7	5.000	0.000	0.000	0.000	0.000	0.000	6	0.333
8	5.000	5.000	0.000	0.000	0.000	0.000	3.000	5
column sums	15.333	16.000	12.000	10.000	6.400	11.200	9.200	5.733

Converge Matrix

Source: Product work of the author.

Figure 102 Calculations part 'b' for test 19.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0412
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0271
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0663
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0190
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.3822
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0984
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1320
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.2339
									###############	

stop delta	0.000005	eigenvalue	8.153573892
delta	0.000005	C.I.	0.021939
power	147	C.R.	0.015671

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	1.523	0.623	3.000	0.108	0.419	0.200	0.200
2	0.657	1.000	0.409	1.000	0.071	0.200	0.205	0.200
3	1.606	2.446	1.000	3.494	0.200	0.673	0.502	0.283
4	0.333	1.000	0.286	1.000	0.050	0.193	0.144	0.081
5	9.267	14.111	5.000	20.157	1.000	5.000	2.896	1.634
6	2.385	5.000	1.485	5.188	0.200	1.000	0.746	0.421
7	5.000	4.872	1.992	6.959	0.345	1.341	1.000	0.333
8	5.000	5.000	3.530	12.334	0.612	2.377	3.000	1.000
	25.249	34.952	14.325	53.133	2.586	11.204	8.693	4.153

RI 0.22947

stop delta	0.000005	eigenvalue	8.153531611
delta	0.000005	C.I.	0.021933087
power	7	C.R.	0.095580264

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	300483.866	467326.770	179937.127	644890.810	31425.164	123148.650	95927.535	54791.065	1897930.986	0.0412
2	197344.511	306919.247	118174.620	423535.509	20638.635	80878.530	63000.869	35984.259	1246476.179	0.0271
3	482719.108	750747.804	289064.054	#######	50483.661	197834.936	154104.887	88020.332	3048974.363	0.0663
4	138153.121	214862.212	82729.475	296500.675	14448.310	56619.910	44104.499	25191.215	872609.416	0.0190
5	#######	#######	#######	#######	291239.632	1141307.291	889029.255	507788.322	17589495.526	0.3822
6	716803.459	1114806.528	429239.523	#######	74964.643	293770.780	228834.922	130703.915	4527509.050	0.0984
7	961457.643	#######	575744.797	#######	100551.087	394038.675	306939.186	175315.068	6072809.570	0.1320
8	#######	#######	#######	#######	178209.859	698367.339	543997.361	310716.198	10763036.236	0.2339
									46018841.327	

Source: Product work of the author.

Figure 103 Schedule for test 20.

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0						1	1
2		0	1		1			
3		1	0					
4				0	1	1		
5		1		1	0			1
6				1		0		1
7	1						0	1
8	1				1	1	1	0

generate s chedule

Unlock

Comparison

1	4	5
2	4	6
3	7	8
4	2	5
5	6	8
6	2	3
7	1	7

Random Redundant Comparisons

8	5	8
9	1	8

vertex	degree
1	1
2	2
3	1
4	2
5	2
6	2
7	2
8	2

#	row	column	length
1	1	7	1
2	2	3	1
3	2	5	1
4	4	5	1
5	4	6	1
6	6	8	1
7	7	8	1
8	1	8	2
9	2	4	2
10	3	5	2
11	4	8	2
12	5	6	2
13	6	7	2
14	1	6	3
15	2	6	3
16	3	4	3
17	4	7	3
18	5	8	3
19	1	4	4
20	2	8	4
21	3	6	4
22	5	7	4
23	1	5	5
24	2	7	5
25	3	8	5
26	1	2	6
27	3	7	6
28	1	3	7

Source: Product work of the author.

Figure 104 Calculations part 'a' for test 20.

Test 20 dark.xls

Adjacency Matrix

Duplicate of original adjacency matrix M1

0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	1	1
2	0	0	1	0	1	0	0	0
3	0	1	0	0	0	0	0	0
4	0	0	0	0	1	1	0	0
5	0	1	0	1	0	0	0	1
6	0	0	0	1	0	0	0	1
7	1	0	0	0	0	0	0	1
8	1	0	0	0	1	1	1	0

Transfer top half

Add Zeroes

Modify shaded cells in top half of this AHP matrix M2

0	1	2	3	4	5	6	7	8	
1	1	0.00	0.000	0.000	0.000	0.000	0.200	0.200	5
2	0.000	1	0.333	0.000	0.200	0.000	0.000	0.000	5
3	0.000	3.000	1	0.000	0.000	0.000	0.000	0.000	6
4	0.000	0.000	0.000	1	0.200	0.200	0.000	0.000	5
5	0.000	5.000	0.000	5.000	1	0.000	0.000	1.000	4
6	0.000	0.000	0.000	5.000	0.000	1	0.000	1.000	5
7	5.000	0.000	0.000	0.000	0.000	0.000	1	5.000	5
8	5.000	0.000	0.000	0.000	1.000	1.000	0.200	1	3

Harker matrix M3

0	1	2	3	4	5	6	7	8
1	6	0.000	0.000	0.000	0.000	0.000	0.200	0.200
2	0.000	6	0.333	0.000	0.200	0.000	0.000	0
3	0.000	3.000	7	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	6	0.200	0.200	0.000	0.000
5	0.000	5.000	0.000	5.000	5	0.000	0.000	1.000
6	0.000	0.000	0.000	5.000	0.000	6	0.000	1.000
7	5.000	0.000	0.000	0.000	0.000	0.000	6	5.000
8	5.000	0.000	0.000	0.000	1.000	1.000	0.200	4
column sums	16.000	14.000	7.333	16.000	6.400	7.200	6.400	11.200

Converge Matrix

Source: Product work of the author.

Figure 105 Calculations part 'b' for test 20.

M4

0	1	2	3	4	5	6	7	8	RS	N RS
1	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0589
2	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0169
3	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0444
4	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.0211
5	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1077
6	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1184
7	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.4838
8	#########	#########	#########	#########	#########	#########	#########	#########	###############	0.1489
									###############	

stop delta	0.000005	eigenvalue	8.147161506
delta	0.000005	C.I.	0.021023
power	106	C.R.	0.015016

Resulting matrix will use Harker estimate for missing comparisons M5

0	1	2	3	4	5	6	7	8
1	1.000	3.479	1.327	2.798	0.547	0.498	0.200	0.200
2	0.287	1.000	0.333	0.804	0.200	0.143	0.035	0.114
3	0.754	3.000	1.000	2.108	0.412	0.375	0.092	0.298
4	0.357	1.243	0.474	1.000	0.200	0.200	0.044	0.141
5	1.828	5.000	2.425	5.000	1.000	0.910	0.223	1.000
6	2.009	6.989	2.666	5.000	1.099	1.000	0.245	1.000
7	5.000	28.564	10.896	22.974	4.493	4.087	1.000	5.000
8	5.000	8.789	3.353	7.069	1.000	1.000	0.200	1.000
	16.235	58.064	22.475	46.753	8.951	8.213	2.038	8.753

		stop delta	0.000005	eigenvalue	8.14664202
RI	0.22947	delta	0.000000	C.I.	0.02094886
		power	8	C.R.	0.091291187

Calculate new eigen vector using Harker values and original values M6

0	1	2	3	4	5	6	7	8	RS	N RS
1	#######	#######	#######	#######	1301964.185	1171084.386	301287.742	1019225.165	24247561.954	0.0589
2	739292.015	#######	894375.023	#######	374169.639	336556.274	86586.643	292913.744	6968471.913	0.0169
3	#######	#######	#######	#######	980748.543	882158.898	226955.137	767766.035	18265294.763	0.0444
4	919356.725	#######	1112212.313	#######	465303.779	418529.164	107676.004	364256.908	8665738.730	0.0211
5	#######	#######	#######	#######	#######	#######	550595.518	#######	44311793.800	0.1077
6	#######	#######	#######	#######	#######	#######	605264.159	#######	48711513.180	0.1184
7	#######	#######	#######	#######	#######	#######	#######	#######	199097412.985	0.4838
8	#######	#######	#######	#######	#######	#######	761233.001	#######	61263875.390	0.1489
									411531662.716	

Source: Product work of the author.

Appendix 6

Completed Data Collection Forms

Figure 106 Test 1 data collection form.

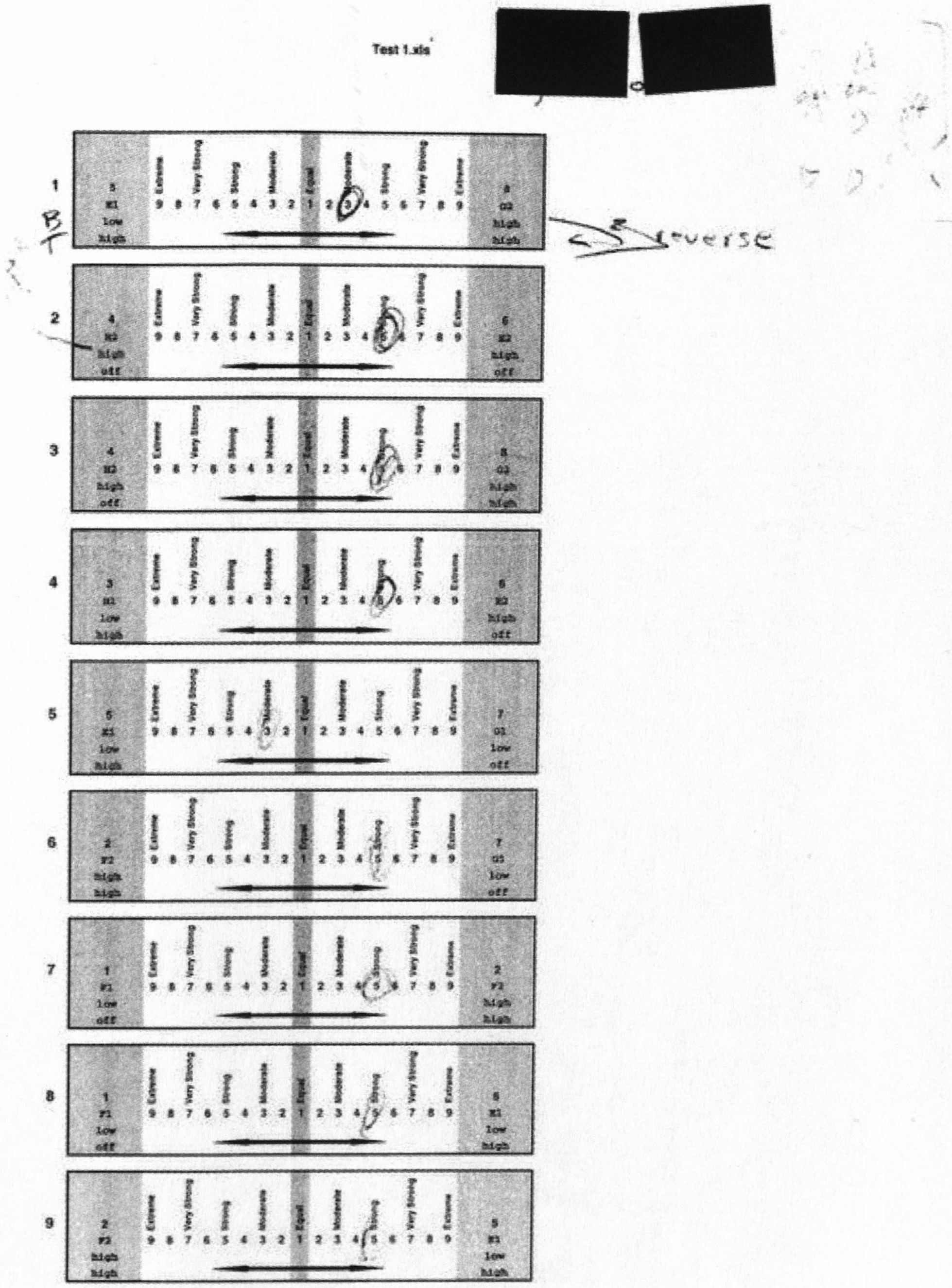
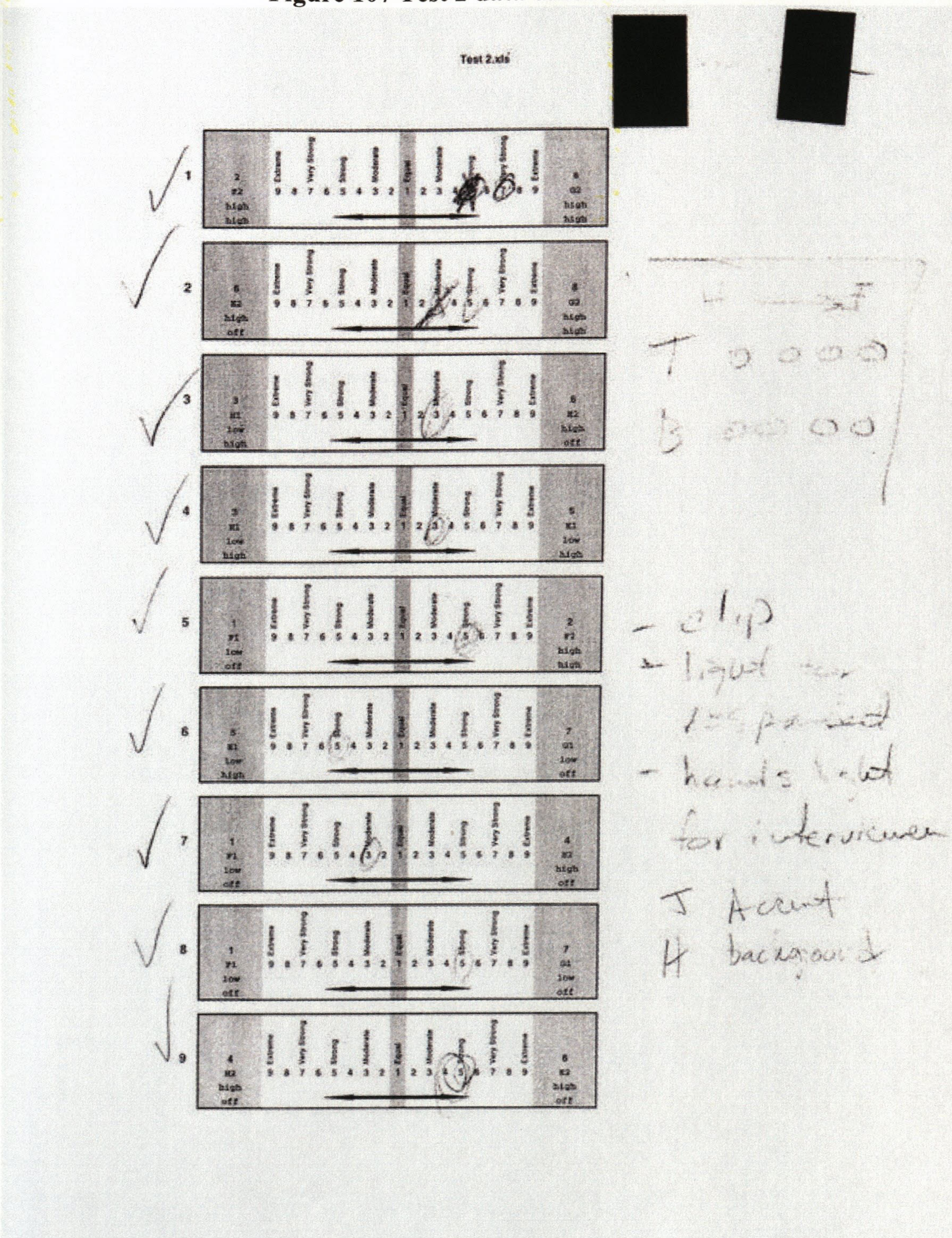


Figure 107 Test 2 data collection form.



Test 2.xls

[redacted] [redacted]

No.	Left	Extreme	Very Strong	Strong	Moderate	Equal	Moderate	Strong	Very Strong	Extreme	Right
		9 8	7	6 5	4 3 2	1	2 3	4 5	6 7	8 9	
1	2 F2 high high										8 G2 high high
2	6 H2 high off										8 G2 high high
3	3 H1 low high										8 H2 high off
4	3 H1 low high										5 H1 low high
5	1 F1 low off										2 F2 high high
6	5 H1 low high										7 G1 low off
7	1 F1 low off										4 H2 high off
8	1 F1 low off										7 G1 low off
9	4 H2 high off										6 H2 high off

T ○ ○ ○ ○

B ○○ ○○

- clip
- liquid for 15 paused
- hands light for interviewer

T Accent

H background

Figure 108 Test 3 data collection form.

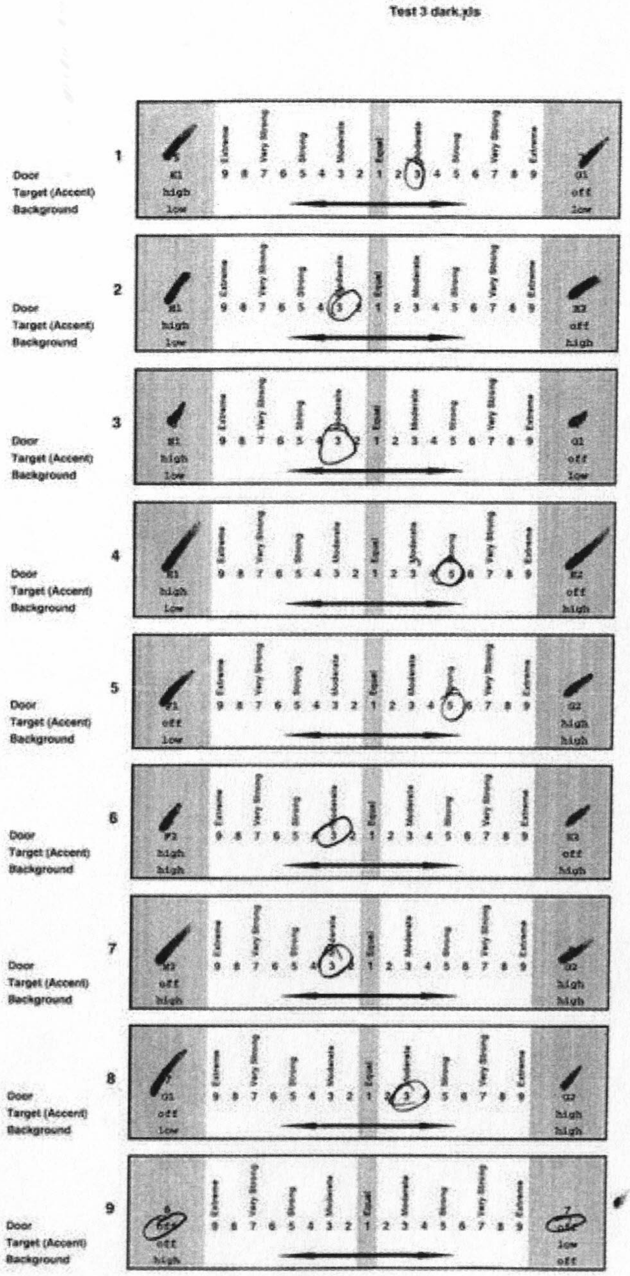


Figure 109 Test 4 data collection form.

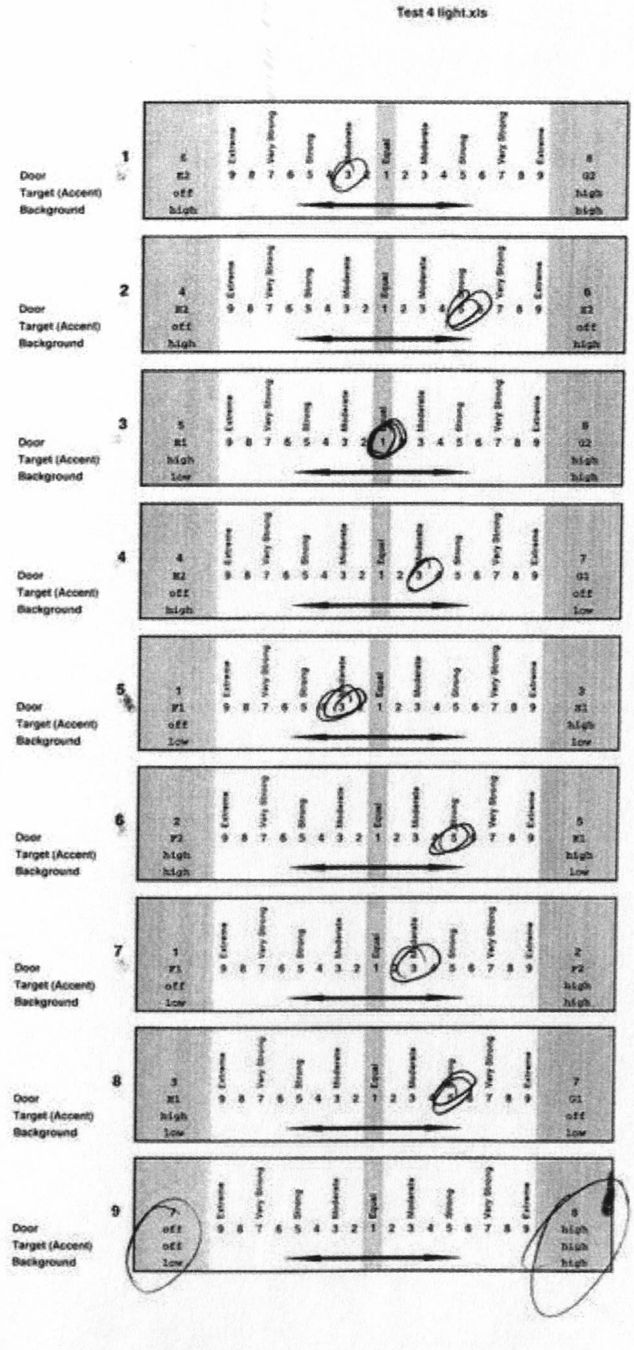


Figure 110 Test 5 data collection form.

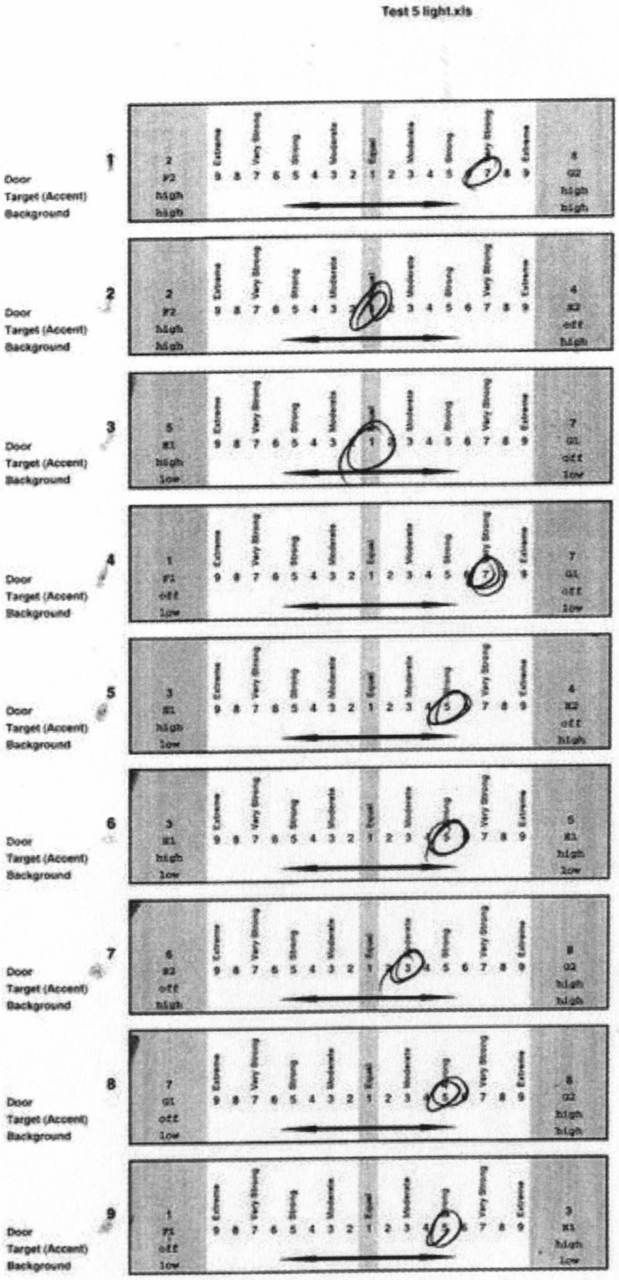


Figure 111 Test 6 data collection form.

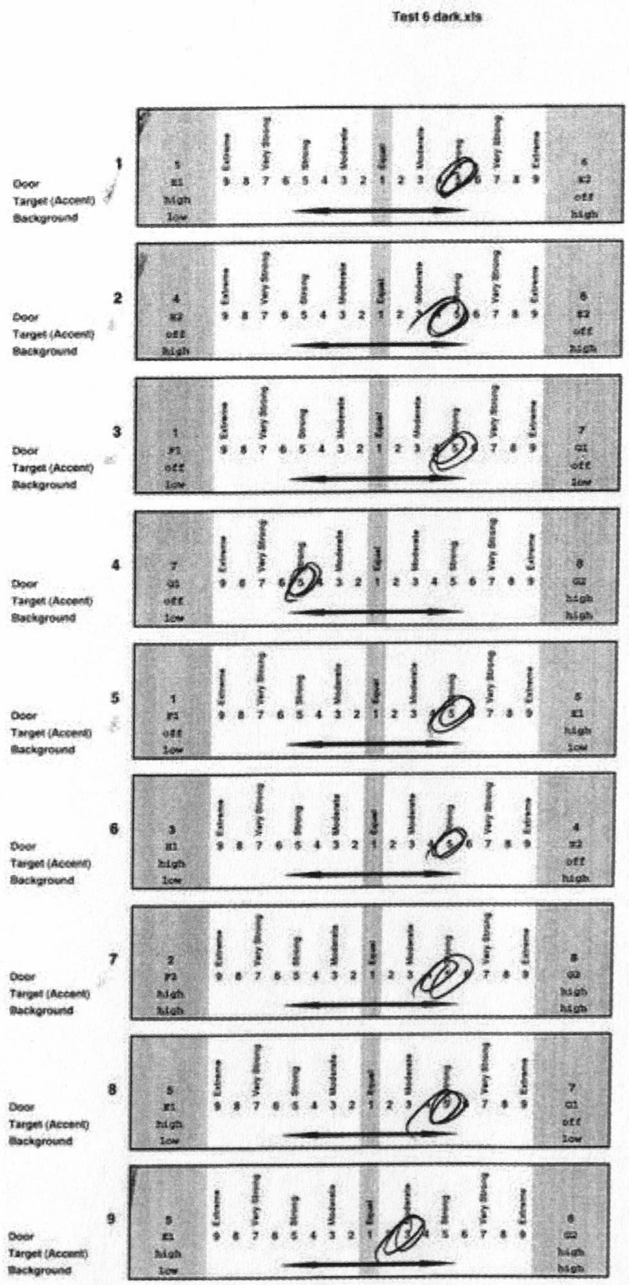
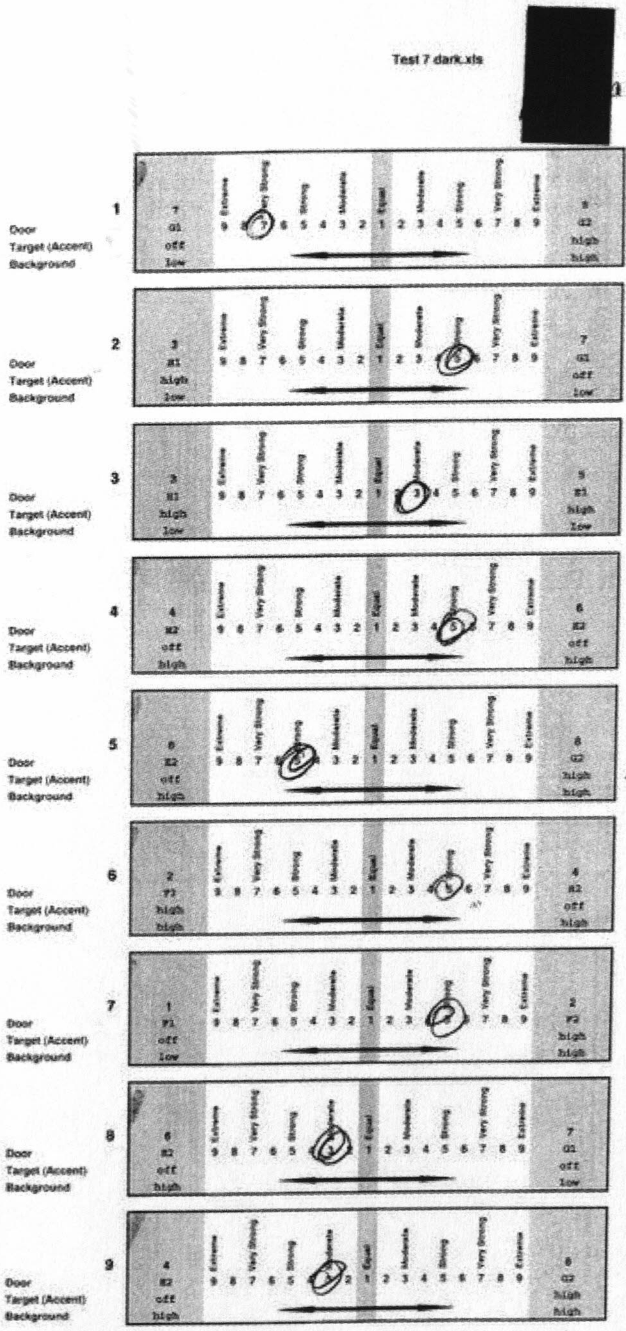


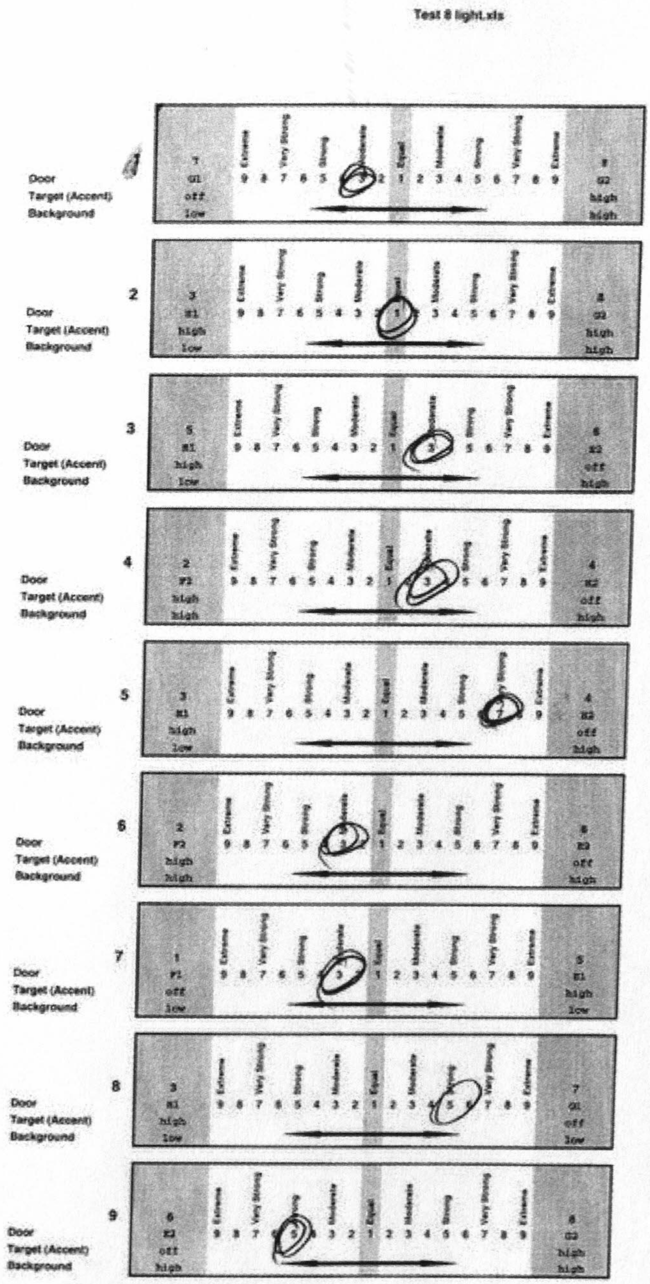
Figure 112 Test 7 data collection form.



[redacted]

- Spots are a negative

Figure 113 Test 8 data collection form.



Does not like dots

Figure 114 Test 9 data collection form.

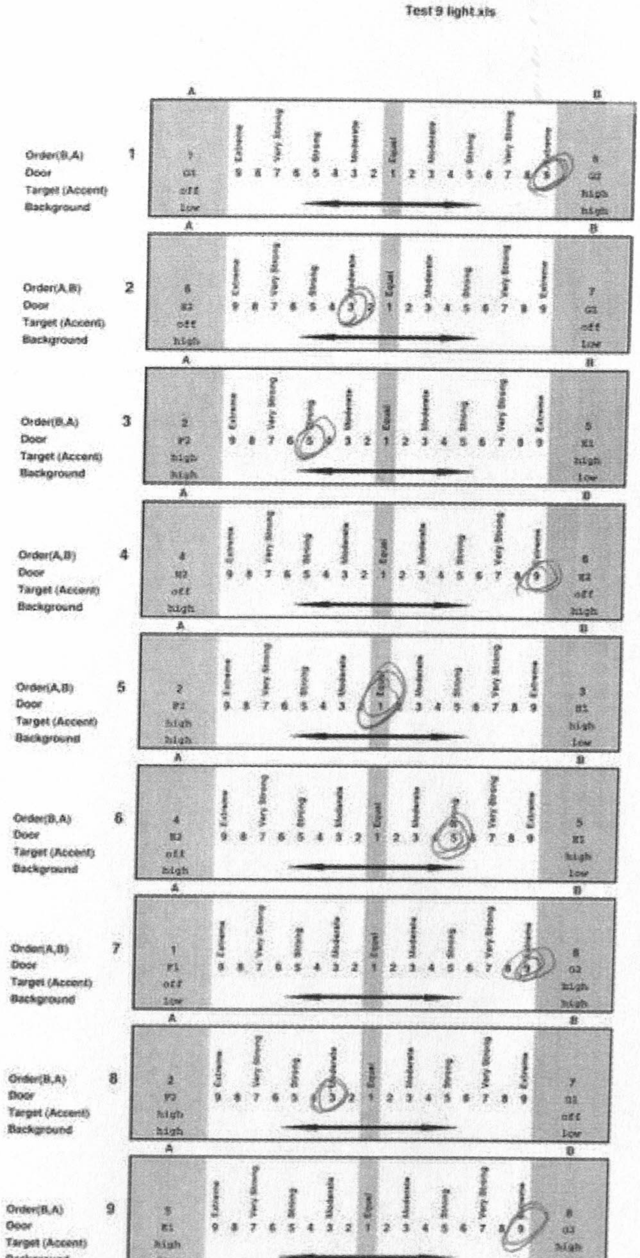


Figure 115 Test 10 data collection form.

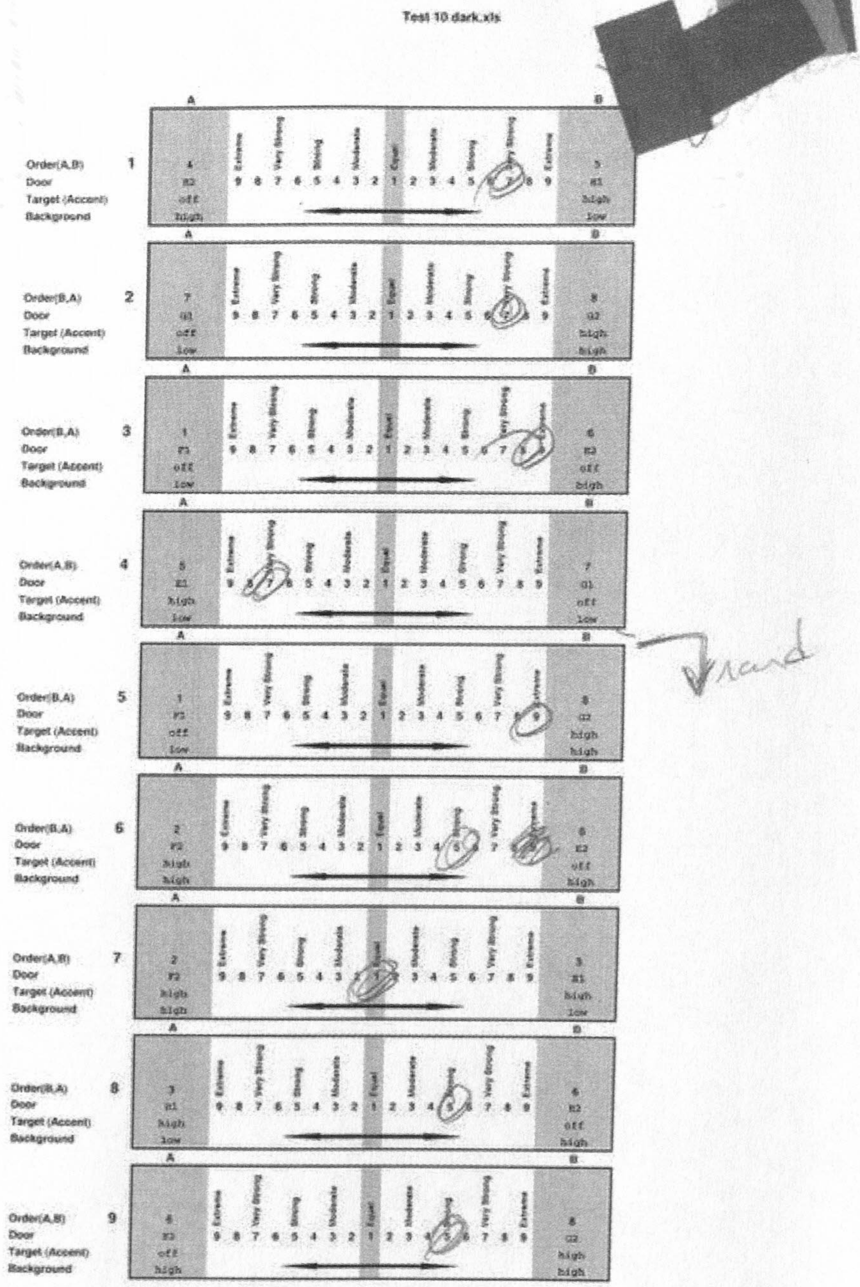


Figure 116 Test 11 data collection form.

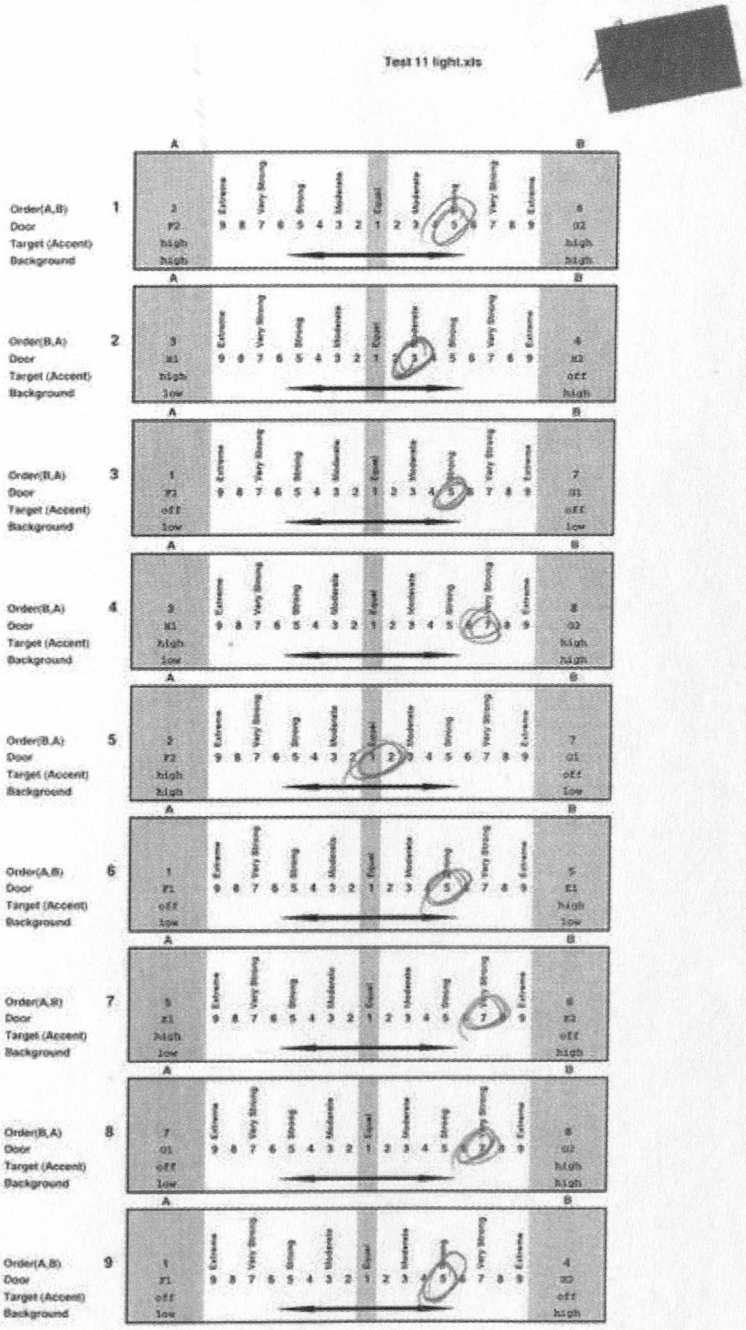
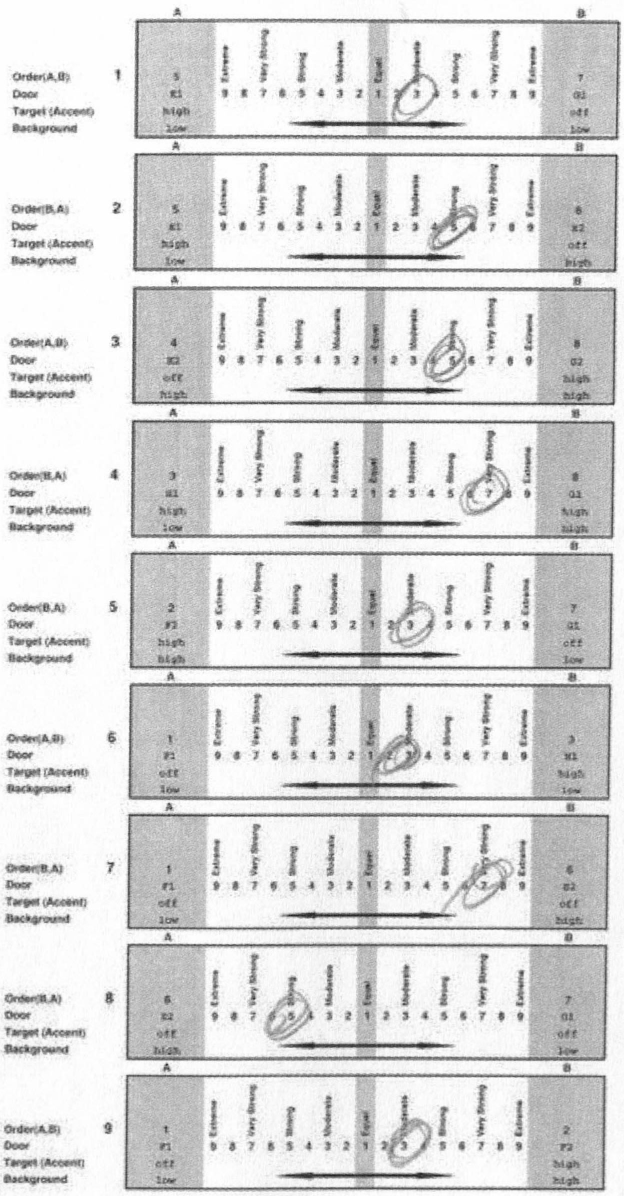


Figure 117 Test 12 data collection form.



Test 12 dark.xls

[redacted]

		A	Scale	B
Order(A,B) / Door / Target (Accent) / Background	1	5 / E1 / high / low	Extreme Very Strong Strong Moderate Equal Moderate Strong Very Strong Extreme; 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	7 / G1 / off / low
Order(B,A) / Door / Target (Accent) / Background	2	5 / E1 / high / low	Extreme Very Strong Strong Moderate Equal Moderate Strong Very Strong Extreme; 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	6 / E2 / off / high
Order(A,B) / Door / Target (Accent) / Background	3	4 / E2 / off / high	Extreme Very Strong Strong Moderate Equal Moderate Strong Very Strong Extreme; 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	8 / G2 / high / high
Order(B,A) / Door / Target (Accent) / Background	4	3 / E1 / high / low	Extreme Very Strong Strong Moderate Equal Moderate Strong Very Strong Extreme; 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	8 / G1 / high / high
Order(B,A) / Door / Target (Accent) / Background	5	2 / F2 / high / high	Extreme Very Strong Strong Moderate Equal Moderate Strong Very Strong Extreme; 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	7 / G1 / off / low
Order(A,B) / Door / Target (Accent) / Background	6	1 / F1 / off / low	Extreme Very Strong Strong Moderate Equal Moderate Strong Very Strong Extreme; 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	3 / E1 / high / low
Order(B,A) / Door / Target (Accent) / Background	7	1 / F1 / off / low	Extreme Very Strong Strong Moderate Equal Moderate Strong Very Strong Extreme; 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	6 / E2 / off / high
Order(B,A) / Door / Target (Accent) / Background	8	6 / E2 / off / high	Extreme Very Strong Strong Moderate Equal Moderate Strong Very Strong Extreme; 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	7 / G1 / off / low
Order(A,B) / Door / Target (Accent) / Background	9	1 / F1 / off / low	Extreme Very Strong Strong Moderate Equal Moderate Strong Very Strong Extreme; 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	2 / F2 / high / high

Figure 118 Test 13 data collection form.

[redacted]

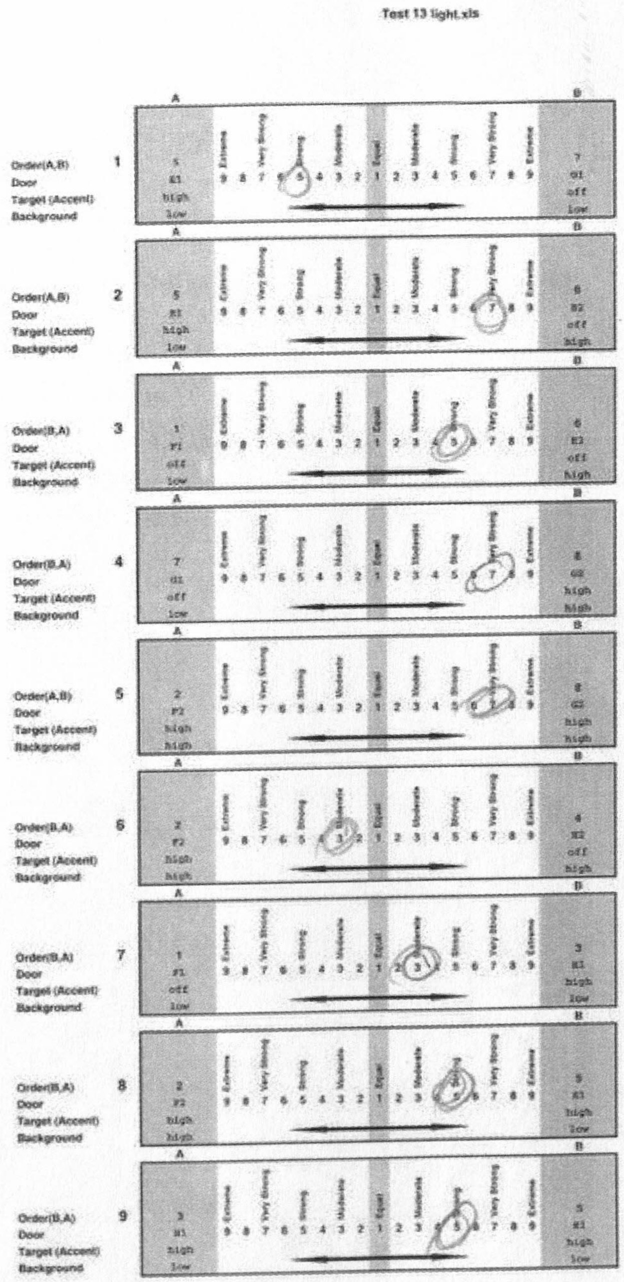


Figure 119 Test 14 data collection form.

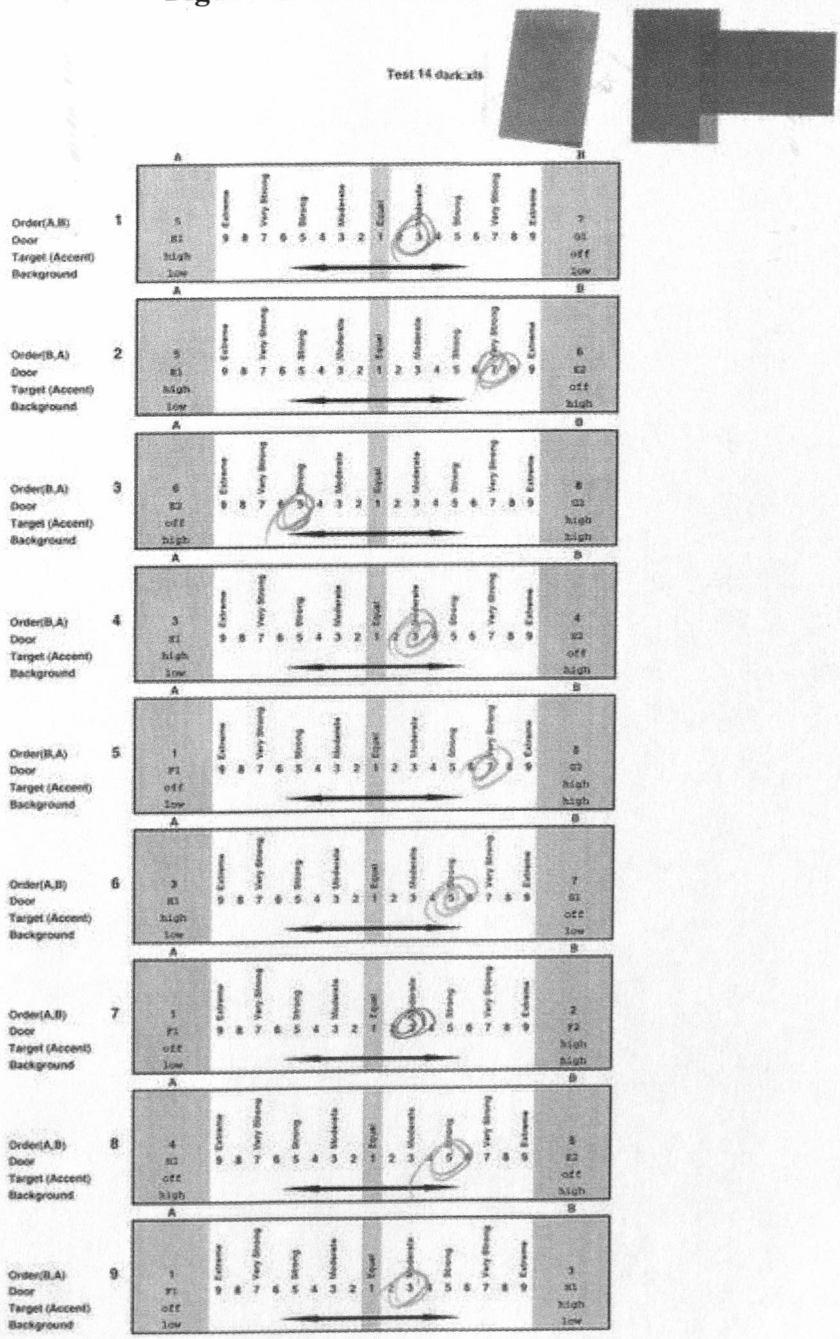


Figure 120 Test 15 data collection form.

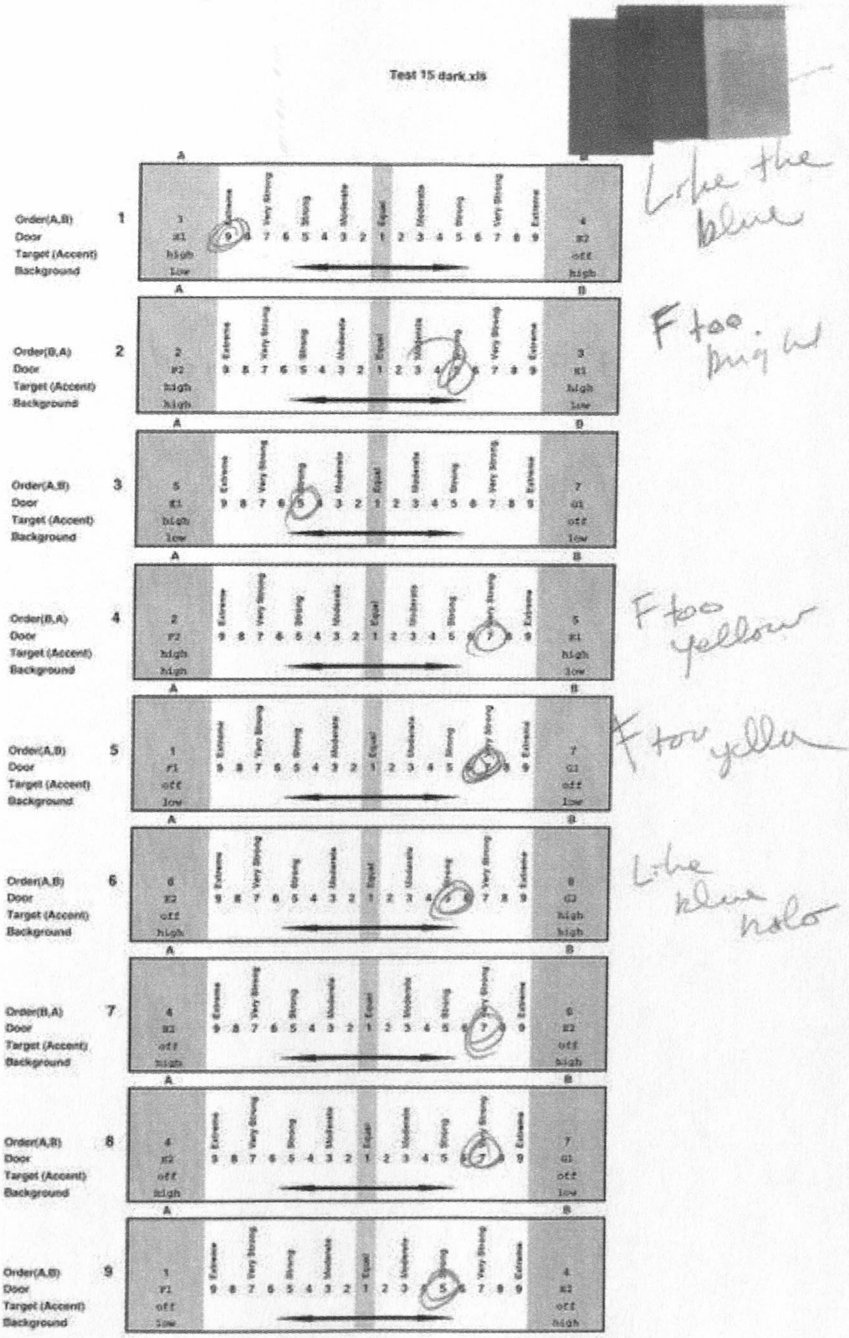


Figure 121 Test 16 data collection form.

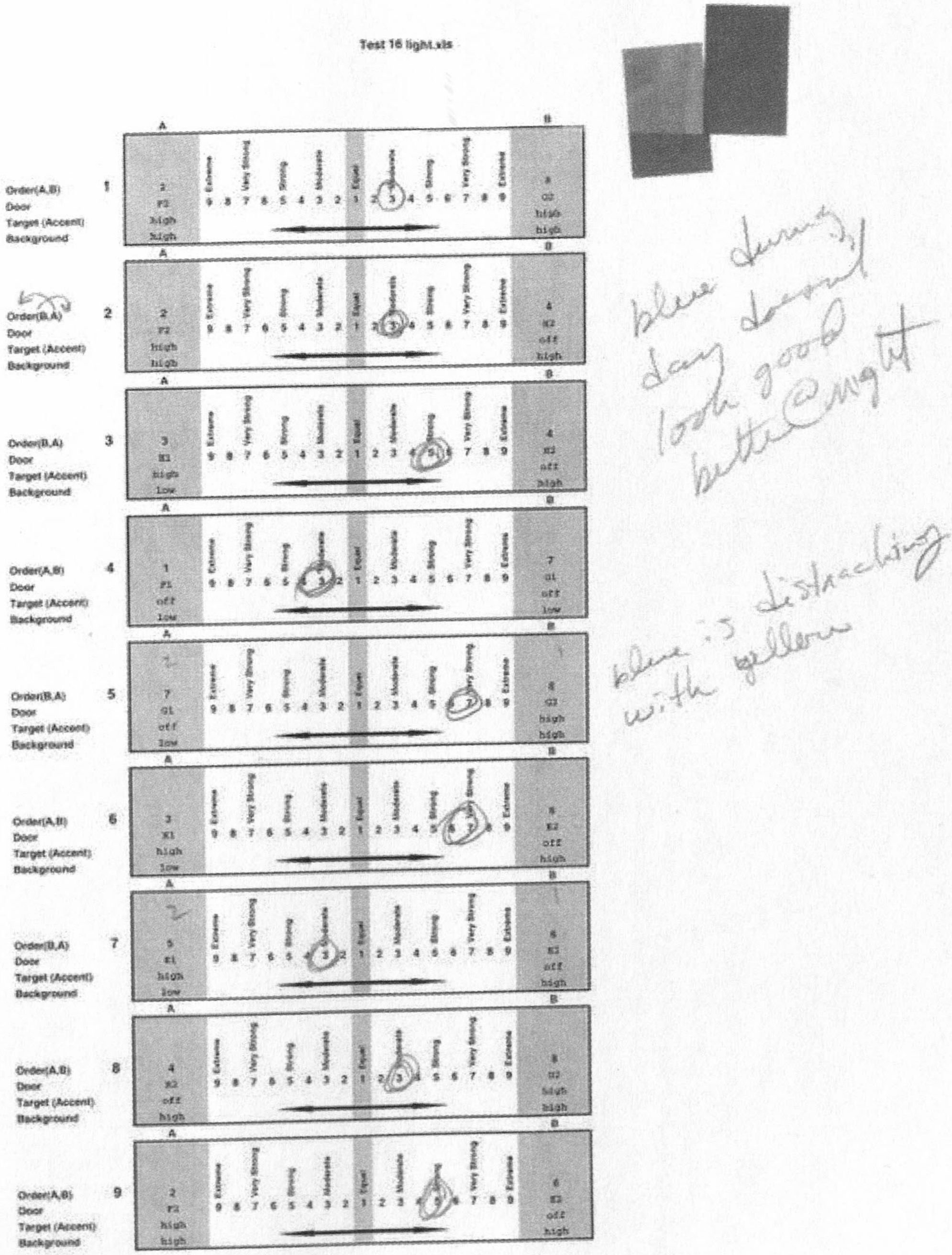


Figure 122 Test 17 data collection form.

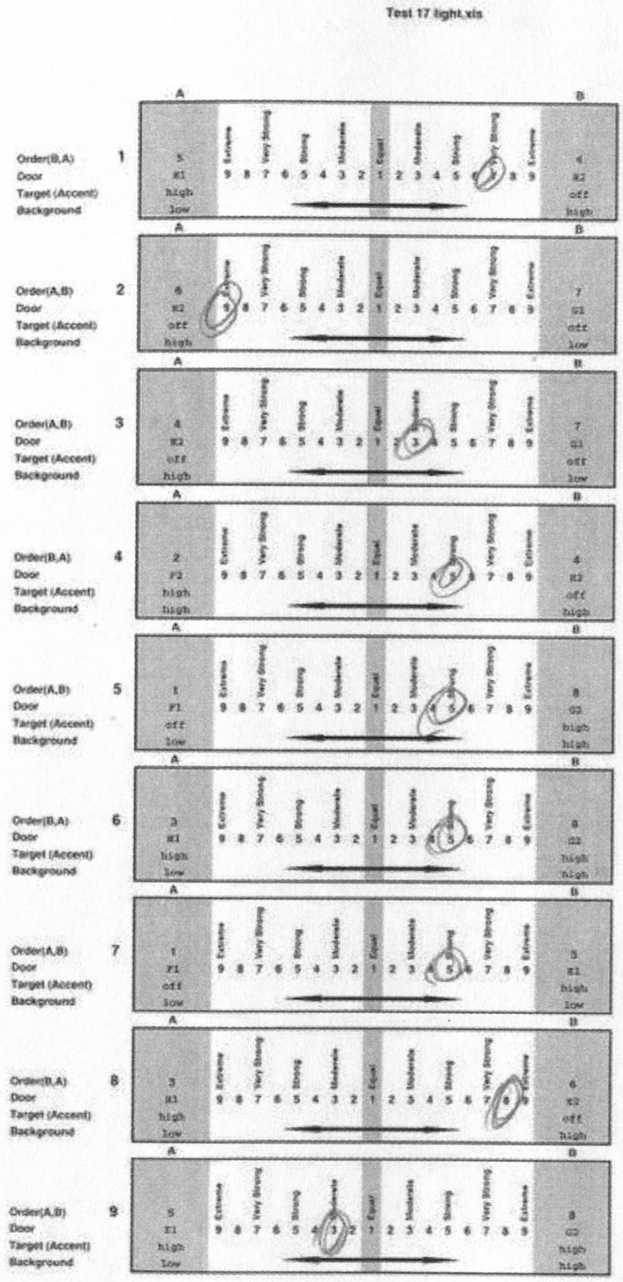
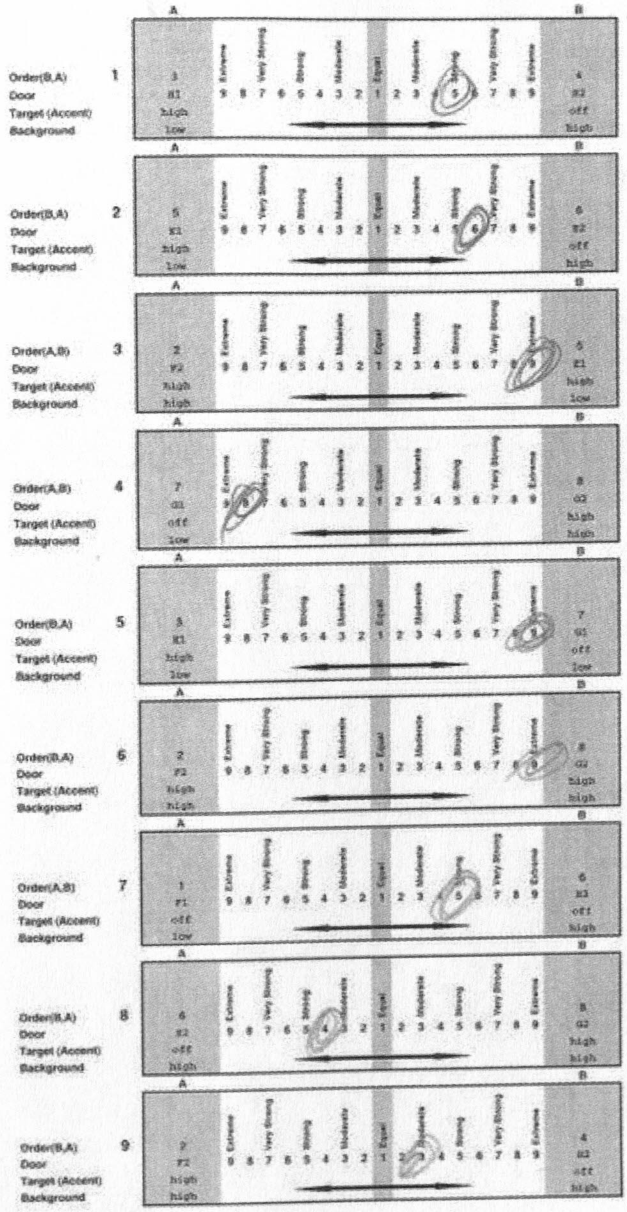


Figure 123 Test 18 data collection form.



Test 18 dark.xls

Scale (A to B): Extreme 9 · 8 · Very Strong 7 · 6 · Strong 5 · 4 · Moderate 3 · 2 · Equal 1 · 2 · Moderate 3 · 4 · Strong 5 · 6 · Very Strong 7 · 8 · Extreme 9

	Order	A	A Door	A Target (Accent)	A Background	B	B Door	B Target (Accent)	B Background
1	Order(B,A)	3	H1	high	low	4	H2	off	high
2	Order(B,A)	5	E1	high	low	6	E2	off	high
3	Order(A,B)	2	F2	high	high	5	E1	high	low
4	Order(A,B)	7	G1	off	low	8	G2	high	high
5	Order(B,A)	5	E1	high	low	7	G1	off	low
6	Order(B,A)	2	F2	high	high	8	G2	high	high
7	Order(A,B)	1	F1	off	low	6	E2	off	high
8	Order(B,A)	6	E2	off	high	8	G2	high	high
9	Order(B,A)	2	F2	high	high	4	H2	off	high

Doesn't like puddle lights

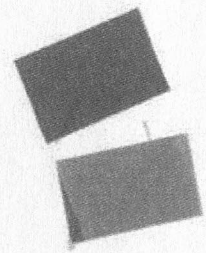
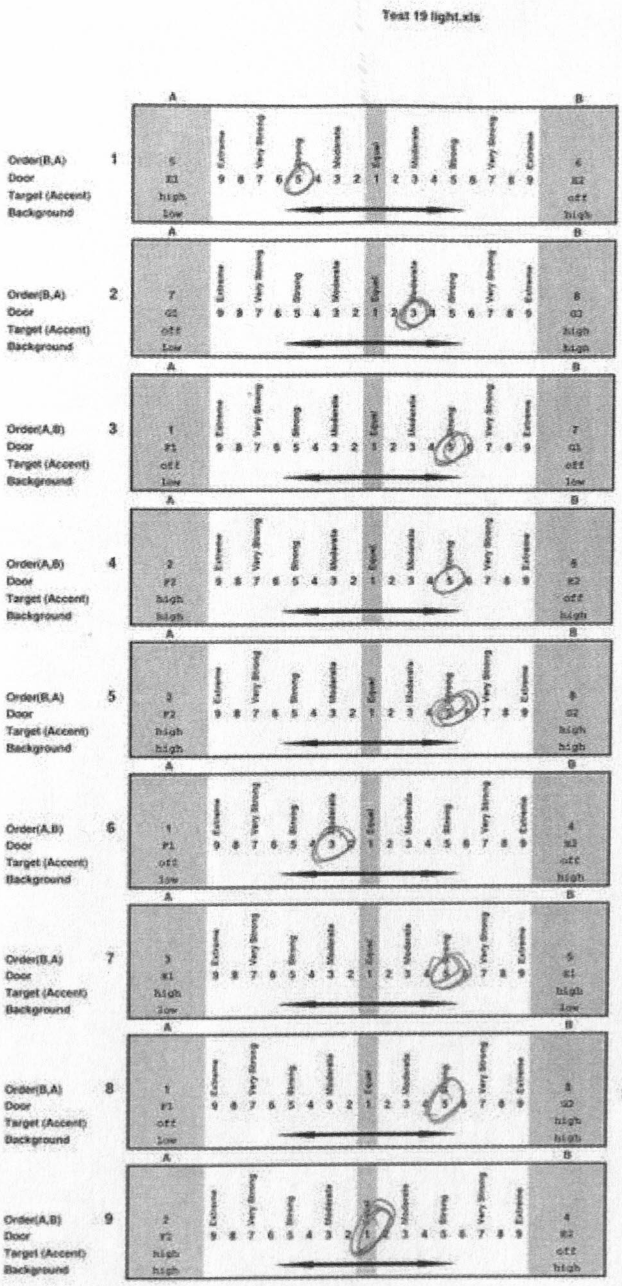


Figure 124 Test 19 data collection form.



don't like yellow

don't like yellow

Figure 125 Test 20 data collection form.